

Multitasking, Limited Liability and Political Agency

John William Hatfield
Graduate School of Business
Stanford University
and Gerard Padró i Miquel*
Graduate School of Business
Stanford University

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Abstract

This paper considers a political accountability framework in which the politician exerts unobserved effort in two independent dimensions. Since the worst payoff to the politician is removal from office, this contracting environment exhibits limited liability. We show that limited liability implies that it is difficult to implement vectors that devote attention to both dimensions. Hence citizens must decide between a high effort allocation to a single task or a low total effort allocation split between the two tasks. Given this, we consider why we do not observe more direct elections of separate ministers, which would allow for better balanced allocations of effort. We find that if elections are primarily used as devices to weed out low type politicians, a united executive dominates one with divided accountability. These results give support to the view that elections act chiefly as selection devices. JEL#: D72, D86

*Corresponding author. E-mail address: gpadro@stanford.edu. 518 Memorial Way, Stanford, CA 94305. Telf: 650.724.0166

1 Introduction

The mandate of the chief executive of a country is very often multidimensional. For instance, in the United States, the President is responsible for economic, social, and foreign policy, and each of these dimensions involves solving many different problems. The citizens care about each of these dimensions, and arguably successes in these tasks are very complementary. Presumably, no citizen would be happy if the economy performs well but the U.S. is successfully invaded by Canada. What does this evident multidimensionality imply for the accountability of political leaders? Why do we observe so many different tasks under the responsibility of one chief executive?

In this paper we set out to analyze these questions using a stylized political agency framework. In this model the politician has to exert unobservable effort in two independent policy dimensions. In each dimension, the observable outcome ex-post is discrete, and can either be a success or a failure. Following the tradition in this literature we assume that the politician perceives some utility when reelected, and that the amount of these rents is beyond the control of the citizenry. The power to deny reelection is the only tool available to the citizen to discipline the politician. The citizen can thus be envisaged as choosing a contract ex-ante that associates a probability of reelection to each potential outcome.¹ Given this setting, it is clear that the optimal voting contract will promise reelection with probability 1 if the politician performs in both dimensions and with probability 0 if she fails in both dimensions. However, what should the citizen do if the politician is successful in one dimension but fails in the other?

The analysis shows that promising reelection with too high a probability when there is a single success—for instance, a success in the economy and a failure in foreign affairs, or the opposite outcome—induces the politician to concentrate her effort in one dimension and forgo the other. The reason is that the additional gain in obtaining two successes becomes too small to merit any effort exertion in the second task.² Hence, to obtain an allocation of effort in which both dimensions receive attention, the citizen faces a constraint on the probability of reelection that he can promise in return for a single success. We call this condition the Concavity Constraint

¹The similarities and differences with the traditional principal-agent framework are discussed at length in Section II.

²The situation is akin to the familiar example of the assistant professor. The department wants her to exert effort both in teaching and in research. However, if the department promises tenure when only one task is successful, the professor will concentrate on just one task.

because it arises from the principal's need to keep the problem of the agent globally concave.

Should the citizen want to implement a focused allocation with positive effort in just one dimension, he can disregard this constraint. These focused allocations are already on the boundary of the feasible set and hence there is no need to make the politician's problem globally concave.³ The citizen can thus extract more effort from the politician in focused allocations. As a consequence, citizens must decide between a high effort allocation to a single task or a low total effort allocation split between the two tasks. If successes in these tasks are complementary enough in the preferences of the citizen, he will have to accept a low level of effort.

The originality of this result stems from the fact that it is imposed by the second order conditions of the politician's problem.⁴ The concavity constraint is a consequence of two features of the model. First, the incapacity to punish politicians. If citizens could punish the politician when she fails to deliver any success, they would not need to reward single successes to obtain effort.⁵ Second, an effort cost function with positive cross-derivative across tasks.⁶ If the effort cost function was additively separable, the politician would naturally choose split allocations of effort and there would be no concavity concerns.

This difficulty in controlling shirking in the presence of multiple tasks depends on the structure of accountability. In particular, if the citizen was able to separately reelect or replace the economic minister and the foreign minister, he would obtain better interior allocations of effort because there would be no need to satisfy the concavity constraint simultaneously in the two dimensions. This begs the question: why do we typically elect a single executive responsible for many tasks?

An answer to this question is provided within the framework of political agency models. In this framework, elections have also been conceptualized as a selection device to weed out politicians with low competency levels.⁷ Introducing types in our model rationalizes the existence

³Obviously, the problem still needs to satisfy the second order conditions for this single dimension. But this is automatic by the convexity of the cost function.

⁴The reduction of total effort as the agent takes on extra tasks is not new to the multitask literature. See in particular Holmstrom and Milgrom (1992) and Dewatripont et. al. (1999). However, in previous models it is the increase in noise generated by extra activities that reduces total effort, a force that is absent from our framework.

⁵As we show below, the concavity constraint is also binding in the multitask version of a limited liability case with unbounded wages. This phenomenon thus transcends political agency models.

⁶Our model is very close to Ting (2002). However, in this paper the effort cost is additively separable across task and hence the second order conditions always hold.

⁷Different interpretations have been given to the types: competence, honesty, and ideological congruence are among the most used. See Besley (2006) for a discussion.

of executives with many disparate responsibilities. In particular, the analysis of the pure selection case reveals the opposite result from the moral hazard case: it is always welfare enhancing to keep tasks together under one politician as this generates more signals of her type. Hence the informational rationale justifies the pervasiveness of unified executives responsible for many tasks.

Indeed, when we explore the combined case in which moral hazard and unobserved types coexist, we find that a united executive generally dominates. Hence the informational justification for multitasking governments is robust to the inclusion of concerns related to unobserved actions. This is true despite an important commitment problem that arises when moral hazard and adverse selection are combined.⁸

Since an overwhelming majority of political systems hold the executive accountable as a whole, our model suggests that voters view the presence of different levels of competence in the politician's pool as an overriding concern vis-à-vis the provision of incentives. This result is thus consistent with Fearon (1999).

The model presented here is related to the multitasking literature in the theory of organizations, which emphasizes the difficulties of contracting in a multidimensional outcome setting. The seminal work on multitasking agency, Holmström and Milgrom (1991), relied on risk aversion, differential observability and misalignment between observable outcomes and the utility function of the principal. In our model agents are risk neutral and observable outcomes are exactly what citizens care about. Hence none of these effects from the previous literature are present and we unveil a novel reason that plagues contracting in multitask environments.

Dixit (1996) presents another theory of incentives in the political arena. In his work, multidimensionality complicates incentive provision because it is associated with the presence of a variety of principals that care differently about the different dimensions. This common agency setting damps incentives because the agent can play the principals against each other. In a similar vein, Ferejohn (1986) showed that distributional concerns among the citizens will allow the politician to escape accountability. The model that we propose here abstracts from conflicts between principals and concentrates on the effects of limited liability in multitasking

⁸The commitment problem was first unveiled in Fearon (1999). On the one hand, ex-ante the principal would like to commit to the voting contract that will induce the politician to exert effort. However, ex-post, he cannot do anything else than act according to the beliefs updated with the information revealed by policy outcomes. This is true even if the importance of types is infinitely small.

environments.

This paper is also related to the literature on the optimal structure of government. Persson and Tabellini (2000) offer a comprehensive and unified view on issues such as presidentialism versus parliamentarianism and their effects on accountability. Dewatripont et al (1999) obtain in a related multitask model with career concerns similar results to ours and have predictions on the optimal structure of public agencies. The career concerns model is appropriate to public agencies but political agency introduces different incentives and feasible contracts. Indeed, Alesina and Tabellini (2003) use these differences in a discussion on whether tasks should be performed by politicians or by bureaucrats. To the best of our knowledge, our model is the first one to extend the usual assumptions in political agency models to a multitask environment and to show the importance of the second-order conditions of these problems.

The remainder of the paper is organized as follows. Next section presents a stylized one-dimensional political agency model and discusses the main assumptions in this literature. Section III extends the model to two dimensions and discusses the binding second-order condition and its role in the non-convexity of the implementable set. The following section presents the pure selection model and shows that united government dominates in that setting. Section V analyzes a model with both underlying types and moral hazard. Finally, section VI concludes.

2 A Simple Unidimensional Model

For the sake of comparison, we first examine the standard unidimensional model of political agency with moral hazard.

The principal is the whole of the citizenry and the agent is a politician that has just been elected. The politician is supposed to exert some effort on behalf of the citizenry. The effort level that the politician exerts, $e \in [0, 1]$, is not observable and thus the citizenry has to condition the rewards to the politician on the final outcome that they perceive. Assume that this final outcome O is dichotomous, $O \in \{G, B\}$. The citizens receive utility V_G , when the outcome is G and V_B when it is B , and $V_G > V_B$. The mapping from effort levels to outcomes is uncertain and is given by $\Pr(O = G|e) = e$, $\Pr(O = B|e) = 1 - e$. The cost of effort is given by an increasing, convex and twice-differentiable function $c(e)$.

The first best of this problem is immediate: the optimal effort level is characterized by $c'(e) = V_G - V_B$. Since $c(e^*)$ is convex the second order conditions are always satisfied. In a traditional contract theory problem, the citizens would set up a wage schedule conditional on the outcome perceived, which in this case would reduce to w_G and w_B . The problem of the citizenry would thus be:

$$\max_{w_G, w_B, e} e(V_G - w_G) + (1 - e)(V_B - w_B)$$

subject to

$$\begin{aligned} c'(e) &= w_G - w_B \\ 0 &\leq ew_G + (1 - e)w_B - c(e) \end{aligned}$$

where the outside option of the politician has been normalized to 0. It is well known that in the case of risk neutral principal and agent with no restriction on payoffs, the first best level of effort is attainable by “selling the shop” to the agent. In other words, the solution of the program above entails making the agent residual claimant of all the benefits that her effort produces: $w_G - w_B = V_G - V_B$ which induces the optimal amount of effort e^* . Since the IR constraint is binding, $w_B < 0$. Hence the optimal contract imposes a liability on the agent if she is not successful.

It is easy to see why the political accountability literature has departed from the usual contract theory assumptions. If this model accurately described the political accountability relationship, the wage differential that the politician should perceive when reelected should be on the order of the total increase in the surplus of citizens that her efforts in providing sound economic policy create. In addition, the politician should pay an enormous penalty if the economic results are disappointing. Neither result is empirically tenable.

As a consequence, the literature on political accountability departs in two fundamental aspects from the setup above. These two aspects are related to the nature of rewards perceived by the agent. In particular, the wage politicians receive while in office is typically below their opportunity cost in the labor market and is a very small part of the rewards they obtain from

reelection.⁹ Their valuation of office must come either from other pecuniary rewards, such as increased wages after their tenure in office, or from some intrinsic non-pecuniary motivation in the form of honor, self-aggrandizement or willingness to contribute to the social good, sometimes referred to as “ego-rents”. Clearly, the citizens do not control any of these elements. As a consequence, starting with Ferejohn (1986), a long list of models make two assumptions that define the political agency literature: first, valuation of office by the politician is taken as given from the point of view of the citizen.¹⁰ Second, these rewards do not come at a direct cost to the principal.

In the typical political agency problem, the total utility of a politician in office takes the following form:

$$R(P_G e - P_B(1 - e)) - c(e)$$

where P_G is the probability of reelection that the citizen promises in case of a good outcome. P_B is defined analogously for a bad outcome and R is the fixed exogenous reward upon reelection referred to above.¹¹ Note that $P_B \geq 0$ which implies that the worst payment that the politician may receive is 0. The agent faces a trade-off induced by the strategies of the principal: she can increase “shirking” today, thereby increasing her immediate payoff, but this reduces her probability of reelection and hence her probability of attaining the future rents associated with it.

The citizens choose the probability of reelection associated with each outcome, P_G and P_B . They solve the following program:

$$\max_{P_G, P_B, e} eV_G + (1 - e)V_B$$

⁹See Diermeier et al. (2004) and Groseclose and Milyo (1999).

¹⁰See, for instance Rogoff and Sibert (1988), Austen-Smith and Banks (1989), Rogoff (1990), Banks and Sundaram (1993,1998), Besley and Case (1995), Ashworth (2005), Smart and Sturm (2004) or Snyder and Ting (2006). Most of these papers add an adverse selection component to the underlying moral hazard. Fearon (1999) discusses the relationship between both informational assymetries. For a stylized version of these models see Persson and Tabellini (2000) or Besley (2006).

¹¹Many of the models cited are infinitely repeated games. In this case, R is the best continuation value for the politician in a subgame perfect equilibrium. In any case the basic trade-off that the politician faces each period remains the same. And from the point of view of the current citizen, these future rents are not a choice variable. Hence making the game repeated does not change the basic incentive structure of the stage game.

subject to

$$\begin{aligned} c'(e) &= (P_G - P_B)R \\ 0 &\leq P_G, P_B \leq 1 \end{aligned}$$

The first constraint is the first order condition of the problem of the politician. The individual rationality constraint is ignored because it will not be binding in a limited liability setting. The solution of this program is straightforward: in the case of a good outcome, reelect the politician, $P_G = 1$. If the outcome turns out bad, oust her from power $P_B = 0$. This scheme widens as much as possible the difference in payoffs between the two outcomes, and since it comes at no direct cost to the citizenry, it extracts the optimal amount of effort. Note that effort levels will be below first best levels as long as $R < V_G - V_B$, which conceptually must surely be the case.

3 Political Agency in a Simple Multidimensional Model

3.1 Environment, Timing and Definition of Equilibrium

The politician can exert effort in two identical tasks on behalf of the citizenry. Denote tasks by the lowercase letters a and b . The outcomes in both tasks can either be good or bad, denoted G and B respectively. The two dimensional outcome space thus has four elements: $(O_a, O_b) \in \{(G, G), (G, B), (B, G), (B, B)\}$. The citizens have preferences defined on the outcome space, $V(O_a, O_b)$, that are characterized by four numbers, V_{GG} , V_{BG} , V_{GB} and V_{BB} . For simplicity we will assume symmetry and scale the values so that $V_{GG} = 1$, $V_{BG} = V_{GB} = \zeta$, and $V_{BB} = 0$. The politician receives exogenous utility R if she is reelected and exerts effort at a cost $c(e_a, e_b) = \frac{1}{2}(e_a + e_b)^2$.¹² As in the previous section, the technology is linear, $\Pr(O_i = G|e_i) = e_i$, for $i = a, b$. Assume further that both the politician and the citizenry are risk neutral.

The timing of the model is as follows:

1. The citizenry presents a voting function to the politician, $P(O_a, O_b) : [G, B] \times [G, B] \rightarrow [0, 1]$. This function maps the outcome space into the probability of reelection. Since

¹²For simplicity, we use the quadratic cost function. However, the results obtained are not exclusive of this functional form. The qualitative results can also be obtained with any cost function of the type $c(e_a, e_b) = \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 + \delta e_a e_b$ for $\delta > 0$ or any function $c(e_a + e_b)$, for $c(\cdot)$ increasing and convex. Details are available from the authors.

each outcome dimension is dichotomous, the function is completely characterized by four numbers: let P_{ij} be the probability of reelection of the politician in state $(O_a, O_b) = (i, j)$.

2. The politician, upon observing the voting function decides how much effort to exert in each dimension.
3. The outcome vector is realized and the politician is reelected with the probability stated in the voting function for that realization. If she is reelected, she receives utility R .

The citizenry propose a voting function, $P_{ij} \in [0, 1], i, j = G, B$ that maximizes their utility given the effort level with which the politician will respond. The strategy of the politician is a selection of effort conditional on the contract offered to her $\sigma(P_{GG}, P_{GB}, P_{BG}, P_{BB}) : [0, 1]^4 \rightarrow [0, 1]^2$ that maximizes her probability of reelection minus her cost of effort in each subgame. There is a proper subgame for each potential voting function that the citizen may choose. The program of the citizen is the following:

$$\max_{e_a, e_b, P(\cdot, \cdot)} \mathbb{E}[V(O_a, O_b) | (e_a, e_b)] \quad (1)$$

subject to the natural constraints on the reelection probabilities

$$0 \leq P_{ij} \leq 1 \quad i = G, B; j = G, B$$

and that the equilibrium efforts are indeed optimal for the agent:

$$(e_a, e_b) \in \operatorname{argmax}_{(e'_a, e'_b) \in [0, 1]^2} \left\{ R \begin{pmatrix} e'_a e'_b P_{GG} + e'_a (1 - e'_b) P_{GB} + \\ (1 - e'_a) e'_b P_{BG} + (1 - e'_a) (1 - e'_b) P_{BB} \end{pmatrix} - c(e'_a, e'_b) \right\} \quad (2)$$

The last constraint (2) states the problem that the politician solves in each subgame. The analysis will concentrate in showing that the implementation of effort allocations in the interior of the unit square is difficult. Let “focused effort allocations” denote effort vectors of the form $(e_a, 0)$, or $(0, e_b)$. Conversely, let “interior effort allocations” denote any effort vectors for which $e_k > 0, k = a, b$.¹³

¹³Note that, as in the previous section, if we allowed the citizenry to offer unbounded payments to the politician, first best would be easily attainable by the classic procedure of “selling the shop” and cashing in the expected

3.2 The Feasible Set

The feasible set of implementable effort allocations can be determined by finding the agent's best response to each vector of reelection probabilities. We can rewrite the agent's problem as:

$$\operatorname{argmax}_{(e_a, e_b) \in [0,1]^2} \left\{ R \begin{pmatrix} e_a e_b (P_{GG} - P_{GB} - P_{BG} + P_{BB}) + e_a (P_{GB} - P_{BB}) + \\ e_b (P_{BG} - P_{BB}) + P_{BB} \end{pmatrix} - \frac{1}{2} (e_a + e_b)^2 \right\} \quad (3)$$

From this expression, we can identify the elements that can make this objective function non-concave. First, note that in addition to the linear returns to each dimension of effort, there is an interaction term between e_a and e_b . This term makes the marginal return to one dimension dependent on the effort exerted in the other one. This term drives the endogenous non-concavity of the objective function and is amplified by the fact that the cost function exhibits a positive cross-derivative.¹⁴ Formally, the determinant of the Hessian of (3) is:

$$\begin{aligned} & -R^2 (P_{GG} - P_{GB} - P_{BG} + P_{BB})^2 + 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB}) \\ & = R(P_{GG} - P_{GB} - P_{BG} + P_{BB}) (2 - (P_{GG} - P_{GB} - P_{BG} + P_{BB})) \end{aligned}$$

This quantity is negative whenever $\Psi \equiv P_{GG} - P_{GB} - P_{BG} + P_{BB} < 0$.¹⁵ If $\Psi < 0$, an increase in e_a reduces the marginal return to exerting e_b . As a consequence, an increase in effort in one dimension lowers the optimal amount of effort along the other dimension. In other words, if $\Psi < 0$, then the objective function is submodular in e_a and e_b and, as a consequence, there can be no maximum in which both dimensions of effort are supplied in strictly positive quantities. Figures 1 and 2 show the shape of the objective function when this condition is or is not respected.

Lemma 1 *The objective function in (3) features an interior maximum in the unit square only if*

$$P_{GG} - P_{GB} - P_{BG} + P_{BB} \geq 0 \quad (4)$$

value ex-ante. This is possible because both principal and agent are risk neutral and moreover the citizenry can make payoffs conditional exactly on the outcomes they care about (and hence there is no room for "distortion" as in Baker (2003)).

¹⁴In the case with $c(e_a, e_b) = \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 + \delta e_a e_b$, it can be shown that the problem with concavity disappears as $\delta \rightarrow 0$.

¹⁵Note that Ψ cannot be greater than two as $0 \leq P_{ij} \leq 1$.

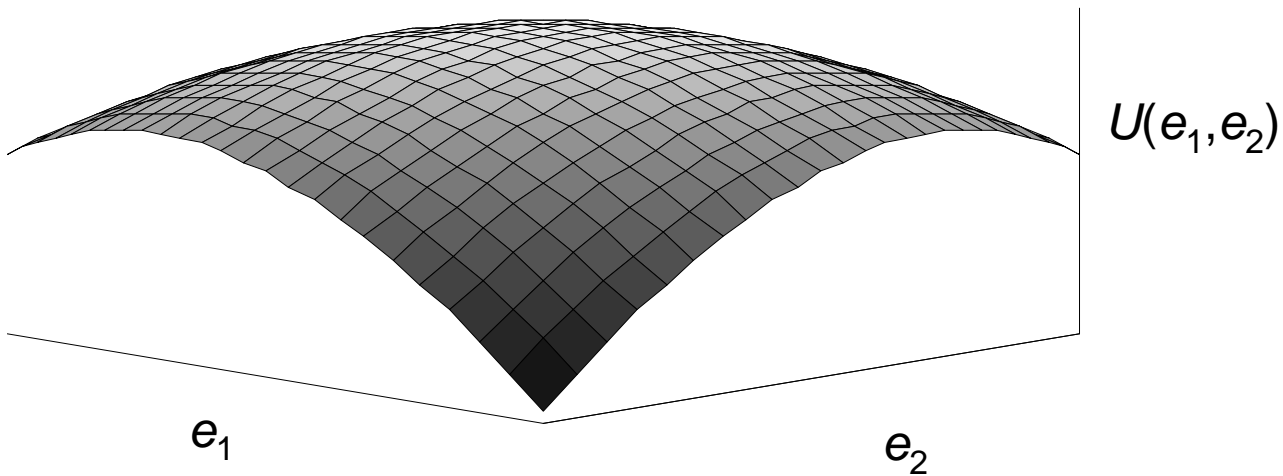


Figure 1: The objective function with $\Psi > 0$

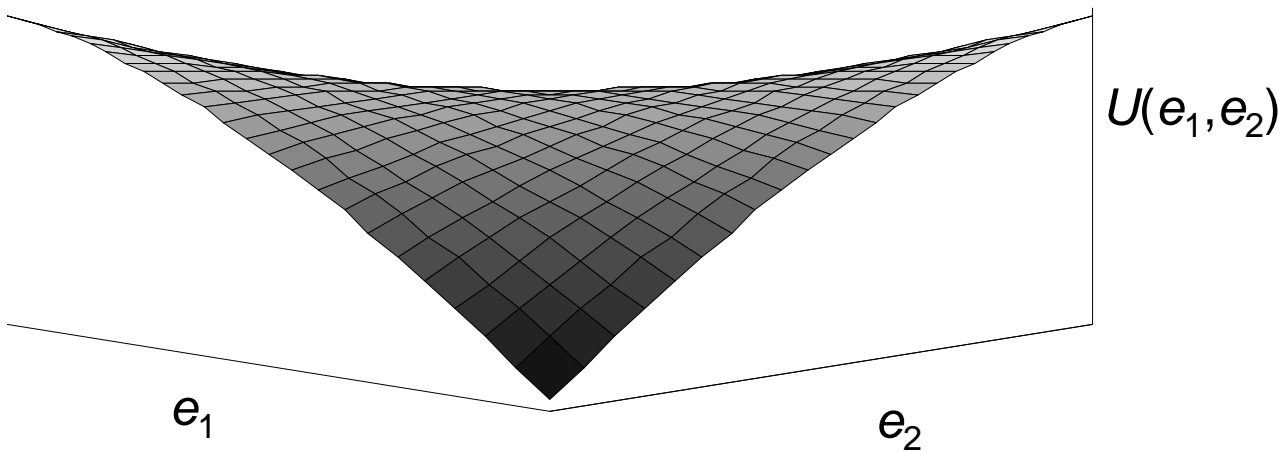


Figure 2: The objective function with $\Psi < 0$

We will call the condition in Lemma 1 the “Concavity Constraint”. Note that for the implementation of an interior allocation of effort, this constraint will have to be satisfied by the principal. The constraint (4) can be read as an upper bound to $P_{GB} + P_{BG}$.¹⁶ Imagine that both P_{GB} and P_{BG} are 0. In this extreme case, the politician will only earn reelection if she obtains two successes. As a consequence, it is obvious that she would exert an interior effort vector. As P_{GB} and/or P_{BG} increase, her returns to concentrating effort in one dimension increase because the prospect of leaving one outcome as a sure failure does not condemn the politician to ejection from power. In the extreme, if $P_{GB} = 1$ the politician has reelection ensured if she provides a success in task a . In such a case, an interior allocation of effort cannot be optimal. This is why the condition for concavity appears as an upper bound to these “cross-diagonal” rewards: to obtain an interior effort vector, the principal has to make sure that he is not rewarding mixed results (a failure in one dimension and a success in the other) too much relative to the reward associated to two successes.

Constraint (4) ceases to be relevant when the citizenry want to implement a focused allocation of effort. In this case the problem only needs to be concave on the one-dimensional space defined by $e_i = 0$, which is ensured by the convexity of $c(e)$. Actually, this case is isomorphic to the problem presented in section 2. The following lemma, which is just a restatement of the result in the previous section, establishes the best focused allocations of effort that the citizenry can induce, given R .

Lemma 2 *The best focused allocation $(e_a, 0)$ is obtained with $P_{GB} = 1, P_{BB} = 0$. Conversely, the best focused allocation $(0, e_b)$ is obtained with $P_{BG} = 1, P_{BB} = 0$.*

The intuition is obvious: if the principal wants the agent to exert maximum e_a , he does so by ensuring that the agent will get maximum rewards whenever outcome a is “good”, and minimum rewards when it is “bad,” irrespective of outcome b . If the principal wants to implement only e_a , O_a is a sufficient statistic, and hence the reward to the agent should be completely independent of O_b .¹⁷ The following corollary pins down the best implementable focused effort vectors.

¹⁶It is easy to show that $P_{BB} = 0$ and $P_{GG} = 1$ in any optimal contract. A formal proof is provided below.

¹⁷See Holmström (1979)

Corollary 1 *The set of best focused allocations is characterized by the pair of points $(e^*, 0)$ and $(0, e^*)$ such that*

$$R = e^*$$

Since R is exogenous and the maximum level of effort in one dimension is technologically bounded above by 1, it will be assumed for the rest of the paper that $R \leq 1$.

To find the boundary of the set of implementable interior effort vectors one has to solve the following program, for $K \in (0, 1)$:

$$\max_{(e_a, e_b, P_{GG}, P_{BB}, P_{BG}, P_{GB}) \in [0, 1]^6} e_a \tag{5}$$

subject to

$$e_b \geq K$$

$$0 \leq P_{ij} \leq 1 \text{ for } i, j = G, B$$

$$\Psi \geq 0$$

$$R[e_b\Psi + P_{GB} - P_{BB}] = e_a + e_b$$

$$R[e_a\Psi + P_{BG} - P_{BB}] = e_a + e_b$$

Note that this program includes the concavity constraint (4) necessary to implement an interior effort vector, as well as the two first order conditions that will determine the effort level in each dimension. Note also that the usual individual rationality constraint is not included in the program because the politician can always guarantee herself utility RP_{BB} by exerting no effort at all.

Proposition 1 *For $R \leq 1$, in the implementation of any optimal interior effort vector:*

i. $P_{GG} = 1$ and $P_{BB} = 0$.

ii. The concavity constraint (4) is always binding.

To help clarify the intuition behind part *i.* in this proposition note that the higher RP_{BB} the more difficult it is to give incentives for effort. Since rewards are bounded above by R , increasing

P_{BB} only reduces the extent to which payoffs can be contingent on performance. Conversely, the principal wants to reward the best signal he has of exertion of effort with the highest reward he can give, because it comes at no cost to him but increases incentives for the politician. Hence, $P_{GG} = 1$.

To understand part *ii.* of proposition 1 note that high values of P_{GB} and P_{BG} have a first order effect in the left hand side of the first order conditions. Hence, keeping them low reduces the marginal return to effort in each dimension. Since the concavity constraint takes the form of an upper bound to these rewards, it is always binding.¹⁸ From the previous proposition, the set of best implementable interior effort allocations can be identified:

Corollary 2 *The set of feasible interior effort allocations is implemented by setting $P_{GB} = P_{BG} = \frac{1}{2}$. It is constituted by all effort vectors such that*

$$\frac{1}{2}R = e_a + e_b$$

with $e_a > 0$, $e_b > 0$.

As it is obvious from the corollary, only the sum of efforts is determined on the boundary. In other words, the set of feasible interior effort allocations that are best for the voters is a segment with negative unit slope.¹⁹

Corollary 1 and Corollary 2 fully characterize the boundary of the feasible set from which the citizenry can choose. It is important to note that this set is not continuous. Note that along the interior boundary, when $e_b \rightarrow 0$, $e_a \rightarrow e^*$. The need to respect the concavity constraint implies that there is a loss of total effort exerted from focused allocations to interior allocations of effort.

Proposition 2 *For a political agency program (1) with exogenously bounded rewards $R \leq 1$, the set of implementable effort vectors is not convex. In particular, any feasible interior allocation*

¹⁸The first order conditions can be solved in closed form. One obtains:
 $e_a = \frac{R[P_{GB}+P_{BG}(R(1-P_{GB}-P_{BG})-1)]}{1-(R(1-P_{GB}-P_{BG})-1)^2}$ and $e_b = \frac{R[P_{BG}+P_{GB}(R(1-P_{GB}-P_{BG})-1)]}{1-(R(1-P_{GB}-P_{BG})-1)^2}$

In this case, when $P_{GB} = P_{BG} = 0$, no effort at all can be extracted from the politician.

¹⁹Note that when the concavity constraint binds, the interaction in the objective function of the agent (1) disappears, leaving only the linear terms. These linear terms have to be rewarded by the same coefficient to prevent the agent from concentrating her effort on the dimension that offers better rewards. This implies that $P_{GB} = P_{BG}$. As a consequence, the politician is indifferent among any vector that respects that sum, and the principal can choose any point in this segment.

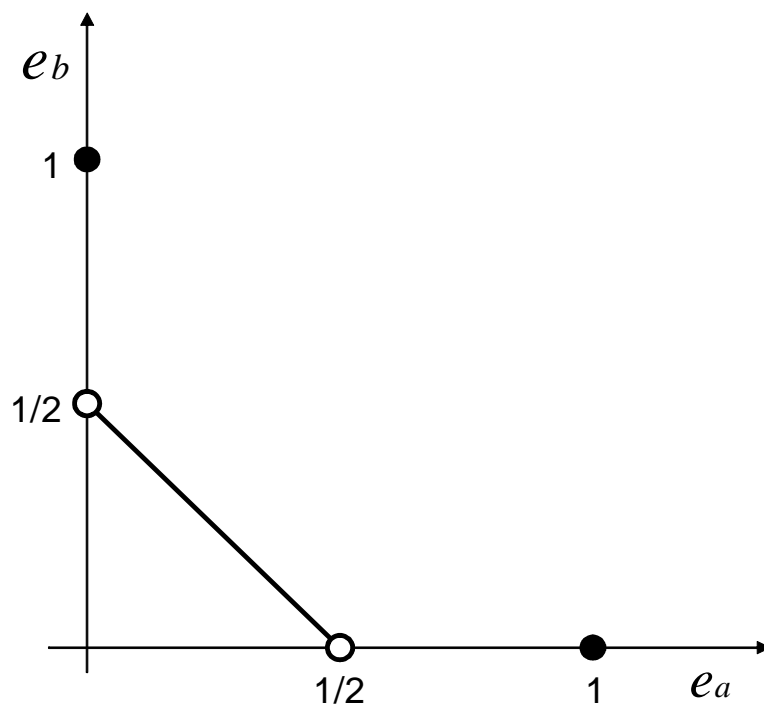


Figure 3: Feasible set with $R = 1$

of effort features less total effort than the best implementable focused allocations.

Figure 3 shows the shape of the feasible implementable set.

3.3 The Principal's Choice

Facing a non-convex choice set, the citizenry has to decide which effort allocation to implement. Two very different alternatives are possible: either the citizens accept a focused allocation in which the politician will completely disregard one of the tasks but work hard at the other, or they try to implement an interior allocation in which both tasks are allocated some effort,

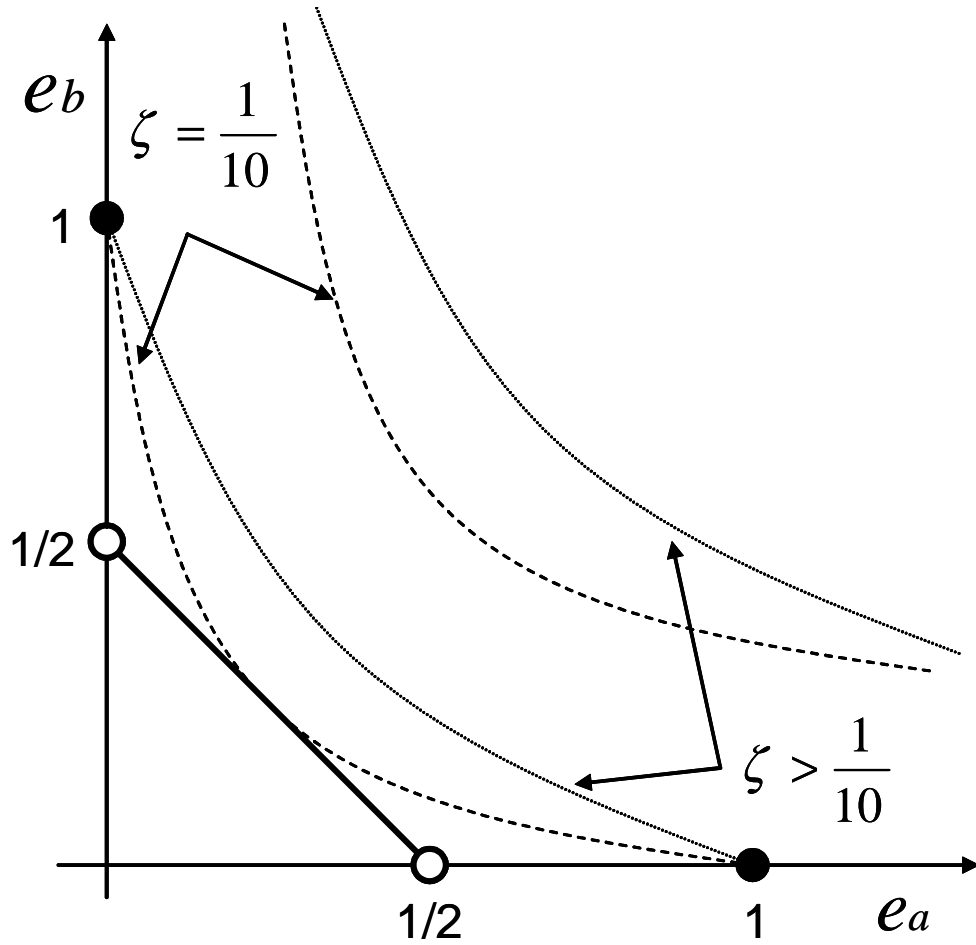


Figure 4: The role of complementarity

but the total effort is much lower. The main determinant of such choice will be the degree of complementarity of the outcomes in the two tasks in the utility function of the principal.

The degree of substitutability in the preferences of the citizenry can be captured by $\zeta \equiv \frac{V_{BG}}{V_{GG}}$. If $\zeta = 0$, outcomes are extremely complementary, because a single success does not provide any utility to the citizen. Intuitively, for low values of ζ , the citizenry will choose the interior allocation. In contrast, for ζ high, the increased effort of the extreme allocation will be chosen. Indeed, it is easy to show that the degree of complementarity necessary for the citizens to forego the effort level of the extreme is very high. In particular, whenever $\zeta > \frac{R}{2(4+R)}$, the citizenry lets the politician concentrate on one task. For example, if $R = 1$, the expression yields $\zeta = \frac{1}{10}$. Figure 4 shows the effect of ζ on the shape of the indifference curves, and consequentially on the optimal contract chosen by the voter.

3.4 Institutional Choice

For small ζ , it is clear that the institutional arrangement that we are describing is not optimal from the point of view of the citizen. In particular, separating accountability of the two tasks makes a much better interior allocation of effort feasible. This is because the Concavity Constraint ceases to be a concern when politicians are responsible for a single outcome.

We can adapt the model in a very straightforward way to introduce this possibility. In particular, assume that prior to the game we described, there is a constitutional stage at which the citizenry decide whether they want a united executive, u or a divided one, d . A divided executive makes separation of accountability possible: two politicians are elected and each one is responsible for a single task and is held accountable separately. For simplicity, assume that each politician perceives rents $\frac{R}{2}$ upon reelection. In a world with only one task, this would result in the same set of feasible effort allocations for the voters. In the world with two tasks, it is immediate to show that the interior allocation of effort $(\frac{R}{2}, \frac{R}{2})$ can be reached. For $\zeta < \frac{1}{2}$, the citizen would prefer this alternative institutional arrangement d . Since complementarity seems an assumption that should be sustained in this political context, we may want to ask why this institutional arrangement is almost never observed. Without entering on matters of economies of scope across tasks, Section 4 offers a novel answer to this question within the framework of political agency.

3.5 Limited Liability

When is the concavity constraint a concern? The political agency literature has departed from traditional contract theory in many ways. Hence we want to ask what particular assumption introduces the binding need to keep the problem concave. In this subsection we show that the crucial assumption is the incapacity to punish the agent typically referred to as limited liability. Limited liability modelling has been extensively used in contract theory.²⁰

To see the importance of limited liability, take the setting of this section and allow the principal to choose the wage in each one of the four potential outcomes. Denote by w_2 the wage associated to two successes, by w_1 the wage associated to one success and by w_0 the wage if no success occurs.

²⁰See Laffont and Martimort (2002) and the references therein.

Note that principal and agent are risk neutral. Hence, first best can easily be reached. However, the optimal contract determines $w_0 < 0$. Limited liability implies a single restriction: no wage can be negative. This constraint is obviously binding. Hence, imposing $w_0 = 0$, the problem of the principal is now:

$$\max_{(e_a, e_b) \in [0,1]^2, w_1, w_2 \geq 0} \{e_a e_b (1 - w_2) + e_a (1 - e_b) (\zeta - w_1) + e_b (1 - e_a) (\zeta - w_1)\} \quad (6)$$

subject to the constraint that the agent is acting optimally:

$$(e_a, e_b) \in \operatorname{argmax}_{(e_a, e_b) \in [0,1]^2} \{e_a e_b w_2 + e_a (1 - e_b) w_1 + (1 - e_a) e_b w_1 - c(e_a, e_b)\}$$

We have:

Proposition 3 *Consider the multitask limited liability program (6). In the implementation of the best symmetric allocation of effort the Concavity Constraint $w_2 - 2w_1 \geq 0$ is binding.*

Since the principal only obtains 1 in the case of two successes, there is a limit to how high w_2 can be. As a consequence, to give incentives, the principal wants to increase w_1 to the point where the concavity constraint binds. This problem would not arise if w_0 could be negative. From the proposition, the solution is determined by the first order conditions and the concavity constraint. In the appendix we show that $w_1 = \frac{\zeta}{\zeta + \frac{3}{2}}$. Hence, as intuition would suggest, all wages are increasing in the returns to the principal. It is easy to show that in this case, the principal will keep the interior allocation as long as $\zeta < \frac{1}{2}$. Again, when tasks exhibit no complementarity, $\zeta > \frac{1}{2}$, the principal gives up on interior allocations and goes for a focused one.

Note again that in this setting with no observability distortions and risk neutrality, none of the reasons that plague multitask implementation in Holmström and Milgrom (1991) or Baker (2003) are present. Dichotomous outcomes and limited liability are enough to create the binding need to keep the problem concave.

4 Pure Selection

In the political accountability literature, elections have also been conceptualized as a device to weed out bad politicians. The basic assumption in this approach is that there are some

underlying types in the pool of politicians. The citizens observe the outcome of the incumbent's term in office, update their beliefs on her type and keep the politician or choose a newcomer according to their posterior beliefs. The framework proposed here can easily be adapted to this selection view of elections.

Take the model of the previous section and maintain the following assumptions: politicians now generate "good" outcomes in the dimensions they are responsible for according to their types $\theta \in \{c, m\}$. The competent type c has the following technology: $\Pr(O_i = G|\theta = c) = q$. On the other hand, the incompetent type m has the poorer technology $\Pr(O_i = G|\theta = m) = s$. We assume $q > s$ and the proportion of competent types in the pool of untried politicians is $\pi < 1$.

The timing of the game is as follows:

1. Nature chooses an incumbent(s) from the pool of politicians.
2. The incumbent(s) generates first period outcomes according to her (their) technology, (O_a^1, O_b^1)
3. Citizens observe the outcomes, update their beliefs according to Bayes' Rule and reelect the politician accordingly.
4. Politician(s) generates the second period outcomes, (O_a^2, O_b^2)

We assume that citizens value outcomes according to $V = V(O_a^1, O_b^1) + V(O_a^2, O_b^2)$. We will compare two institutional structures, divided accountability d or joint accountability u .

In the previous section the framework was implicitly a two period model. However, the second period was irrelevant because it was impossible to extract any effort from the politician. Now the second period gains relevance: by selecting the competent politicians, citizens can increase their expected utility in the second period. Note that the use of elections is conceptually very different: in the moral hazard case, citizens are indifferent ex-post and can thus choose the voting function that will maximize effort extraction for the first period. In the selection case citizens update their beliefs about types and are not indifferent when they decide whether to reelect. Moreover, the return to their voting decision comes in the second period.

Denote by $\mu(O_a^1, O_b^1)$ the posterior belief that the politician's type is c in the case of united executive. We have:

Lemma 3 *Voter's beliefs evolve in the following way:*

- i. If $q + s < 1$, $\mu(G, B) = \mu(B, G) > \pi$*
- ii. If $q + s = 1$, $\mu(G, B) = \mu(B, G) = \pi$*
- iii. If $q + s > 1$, $\mu(G, B) = \mu(B, G) < \pi$*

These posteriors directly imply the electoral behavior of the citizens for the case of united executive. If $q + s < 1$ then good outcomes are relatively difficult to obtain, and a single success is enough to update in the direction of a competent incumbent. Hence the voting function will be $P_{GG} = P_{GB} = P_{BG} = 1$ and $P_{BB} = 0$. On the contrary, when $q + s > 1$, it is easier to produce successes and hence two successes are needed for reelection. In this case, $P_{GG} = 1$ and $P_{BB} = P_{GB} = P_{BG} = 0$.

The case of divided government is much simpler. With a single signal of the politician's type, the citizen cannot do anything different than reelect when there is a success and oust in the case of failure.

Now we can proceed to make welfare comparisons across institutional settings. Denote by $V_j(t)$ the unconditional expected utility of the voter in period $t = 1, 2$ and institutional structure $j = u, d$. Let also $V_j = V_j(1) + V_j(2)$. We now can state the following proposition.

Proposition 4 *In the pure selection case with complementarity, i.e. $\zeta \leq \frac{1}{2}$, $V_u(t) > V_d(t)$ for $t = 1, 2$. Citizens choose united executive for all $\pi \in (0, 1)$ and for all $0 < s < q < 1$.*

By pooling tasks under a single politician, citizens obtain two independent signals of the type of the incumbent which increases their ability to identify types. This is what we will call the selection effect.

In addition, given the technology in the model, united government makes it easier to reach the double success outcome but makes it more difficult to obtain a success and a failure. As long as outcomes are complementary, unconditional utility is higher with united government. In particular one can write $V_u(1) - V_d(1) = \pi(1 - \pi)(q - s)^2(1 - 2\zeta)$. The $1 - 2\zeta$ term captures the gains of reaching GG and BB more often under united government. As long as there is

complementarity, i.e. $\zeta \leq \frac{1}{2}$, the gains outweigh the losses. We call this effect in the first period the technological effect.

In a nutshell, having a united executive can be rationalized by this agency model on informational grounds, even when considerations of scope are sidestepped. Both in the first and second periods there are gains of holding the executive accountable as a whole. This result shows that the world of selection and the world of moral hazard prescribe opposite institutional structures.

5 Multitask Selection and Moral Hazard

Most relationships between politicians and citizens are unlikely to belong entirely to the hidden action or the hidden types paradigm. Rather, both informational issues may be present. We have shown that when the hidden action problem occurs, the citizens prefer a divided executive, while in the unique presence of hidden types, a united executive is preferable. In this section we show that this dominance of united executives is robust to the presence of hidden action concerns.

Note that adding even an infinitely small concern about types has a drastic effect on the citizen's ability to commit to a voting function *ex ante*. In particular, in the interim, first period outcomes are already revealed and hence citizens only care about their second period utility. Since the effort provided by the politician is always 0 in the second period (because the world ends afterwards), citizens cannot commit to do anything different than acting according to Bayes' rule and reelect the incumbent if and only if their posterior belief about her ability is better than the untried pool. This point was made clear by Fearon (1999) in his exploration of the unidimensional case.

We maintain the framework of the third section and add the presence of types as in the fourth section. In particular, assume that the new technology is as follows, $\Pr(O_i = G|\theta = c, e_i) = q + e_i$ and $\Pr(O_i = G|\theta = m, e_i) = s + e_i$. We also assume $q > s$ and the proportion of competent types in the pool of untried politicians is $\pi < 1$. For simplicity and to reduce the number of cases, we will examine the case of pure complementarity, $\zeta = 0$. In addition, we assume that there is symmetric learning in the sense that the politician does not know her own type.

An equilibrium is an allocation of effort (e_a^*, e_b^*) , together with a voting rule $P_{ij} \in [0, 1], i, j =$

G, B such that the politician(s) is(are) acting optimally given the voting rule and the voting rule maximizes the expected welfare of the principal in the second period (since the principal votes after first period payoffs have been realized). When the executive is divided, a single equilibrium exists with a very simple form. The strategies that the posterior beliefs on types prescribe are exactly the same that would maximize effort extraction. Hence, even though the voters suffer from the same inability to commit in both united and divided government, in the latter case this is not a concern.

Proposition 5 *For the divided executive with moral hazard and underlying types, a single equilibrium exists in which:*

- i. $P_G = 1, P_B = 0$ for both tasks, and*
- ii. $e_i^d = \frac{R}{2}$ for $i = a, b$*

The case of united executive is more involved. Denote by $\mu(O_a^1, O_b^1 | e_a^*, e_b^*)$ the posterior belief that the politician's type is c in the case of united executive, given the equilibrium level of effort (e_a^*, e_b^*) . Now we can state the lemma.

Lemma 4 *Voter's beliefs evolve in the following way:*

- i. If $q + s + e_a^* + e_b^* < 1$, $\mu(G, B) > \pi$ and $\mu(B, G) > \pi$*
- ii. If $q + s + e_a^* + e_b^* = 1$, $\mu(G, B) = \mu(B, G) = \pi$*
- iii. If $q + s + e_a^* + e_b^* > 1$, $\mu(G, B) < \pi$ and $\mu(B, G) < \pi$*

Based on these patterns of updating, we can construct two different types of stable equilibria. The next two subsections take them one at a time and compare their properties to the divided executive equilibrium.

5.1 Unbalanced equilibrium

Based on part *i.* of Lemma 4 one can construct an equilibrium in which the voting function will be $P_{GG} = P_{GB} = P_{BG} = 1$ and $P_{BB} = 0$ as long as $q + s + e_a^* + e_b^* < 1$. This voting function, denoted by \tilde{P} , is imposed on the voters because of their inability to commit. Facing \tilde{P} , we can state the problem of the agent as:

$$\max_{(e_a, e_b) \in [0,1]^2} R \mathbb{E}_\theta [(e_a + \theta)(e_b + \theta) + (e_a + \theta)(1 - (e_b + \theta)) + (1 - (e_a + \theta))(e_b + \theta)] - \frac{1}{2}(e_a + e_b)^2 \quad (7)$$

Note that \tilde{P} does not respect the concavity constraint. As a consequence, the politician concentrates her effort on one of the tasks. This makes solving (7) very easy. The equilibrium takes the following form:

Proposition 6 *For $q + s + R(1 - \mathbb{E}[\theta]) < 1$, the following two equilibria exist for the united executive with moral hazard and underlying types:*

- i. Voters use \tilde{P} as their voting function*
- ii. Politicians expend either $(R(1 - \mathbb{E}[\theta]), 0)$ or $(0, R(1 - \mathbb{E}[\theta]))$*

This equilibrium has a number of non-desirable properties from the point of view of the citizen. First, since we are examining the case of pure complementarity, the fact that the politician focuses on a single task is costly. However, the citizen cannot do anything about it because he cannot commit to provide a voting function that satisfies the concavity constraint. In addition, even though every player knows that effort is focused, total effort is inferior to the one in absence of types established in Lemma 2. This is true because the voter cannot commit to oust the ruler if there is a success in the task in which it is known that no effort is devoted.

This unbalance in effort has some consequences for welfare in the first period. In particular, the misallocation of effort causes $V_u(1)$ to increase slower than $V_d(1)$ in R . For R high enough, $V_u(1) - V_d(1) < 0$. The evolution of welfare with respect to R is interesting because keeping q and s fixed, increasing R increases the relative importance of effort extraction vis-à-vis type selection. In particular, when $R = 0$, the model is isomorphic to the pure selection case, and then we know that $V_u(1) - V_d(1) > 0$ because of the technological advantage.

Second, unbalanced effort also affects second period welfare. Recall that the other advantage of a united executive is that it allows better selection and thus higher second period welfare. This remains true when moral hazard is added to the model. However, in this equilibrium with unbalanced effort this advantage is dampened. The reason is that as R increases and more effort is put in one task, the signal from the other task is used less often to decide the electoral outcome.

This is again a consequence of the absence of commitment. This informational externality reduces the ability to select and hence the advantage that united government provides in the second period. The following proposition summarizes these comparative statics.

Proposition 7 *Voters' welfare present the following comparative statics with respect to R :*

- i.* $\frac{\partial V_d(1)}{\partial R} > \frac{\partial V_u(1)}{\partial R} > 0$
- ii.* $\frac{\partial V_u(2)}{\partial R} < 0$
- iii.* $\frac{\partial V_d(2)}{\partial R} = 0$

Hence, as R increases effort and becomes relatively more important, divided government becomes more attractive both in the first and the second period. When $R = 0$, a united government dominates both in the first and second period. $V_u(1) - V_d(1)$ eventually becomes negative when the incentives for effort become large enough, and for yet larger R divided government is better for the citizens than united. Figure 5 shows how citizen's welfare evolves with R and the two institutional arrangements. Note that in every case V_d is steeper than V_u . Also, note that the difference between V_u and V_d at $R = 0$ is increasing in q . This is not surprising because the technological advantage $\pi(1 - \pi)(q - s)^2$ is increasing in q . Finally note that the R at which divided government becomes optimal is increasing in q . As long as types are important (i.e. q is high vis-à-vis R), united government is preferable despite the non-desirable features of the united equilibrium. Hence, the two polar cases explored in the previous sections are linked in a continuous manner when the unbalanced equilibrium is played in the case of a united executive: as long as the moral hazard problem is not dominant, united government is preferable despite the undesirable features that commitment problems introduce in this equilibrium.

5.2 Balanced Equilibrium

Part *iii.* of Lemma 4 provides the foundation for another type of equilibrium to be played in the case of united executive. If $q + s + e_a^* + e_b^* > 1$, citizens respond with the voting function $P_{GG} = 1$, $P_{GB} = P_{BG} = P_{BB} = 0$. Denote this voting function by \hat{P} . In this case, good outcomes are easy to obtain and hence the citizens only update upwards if both tasks are successful. Note that in this case the voting function clearly satisfies the concavity constraint and hence the citizens can

Welfare

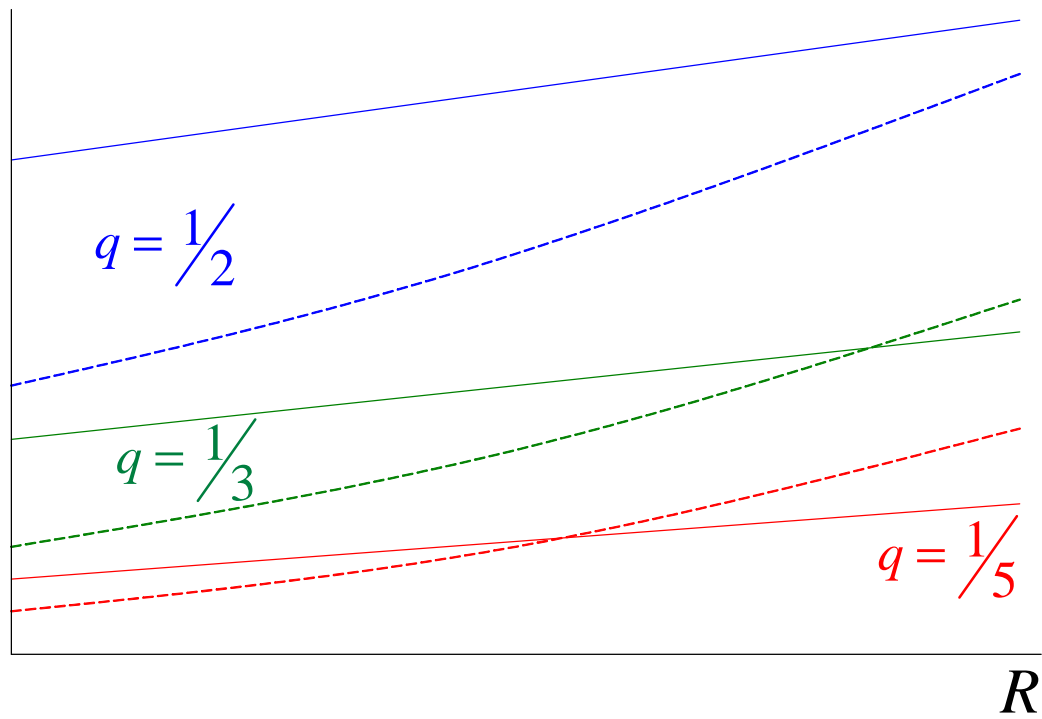


Figure 5: These simulations use $s = 0$ and $\pi = \frac{1}{2}$ and three different values of q . The solid line is V_u while the hashed line is V_d

obtain a symmetric effort vector from the politician. The problem of the agent is now:

$$\max_{(e_a, e_b) \in [0,1]^2} R\mathbb{E}_\theta [(e_a + \theta)(e_b + \theta)] - \frac{1}{2}(e_a + e_b)^2$$

The symmetric solution to this program is immediate, $e_a^* = e_b^* = \frac{R\mathbb{E}[\theta]}{2-R}$. This establishes the following proposition.

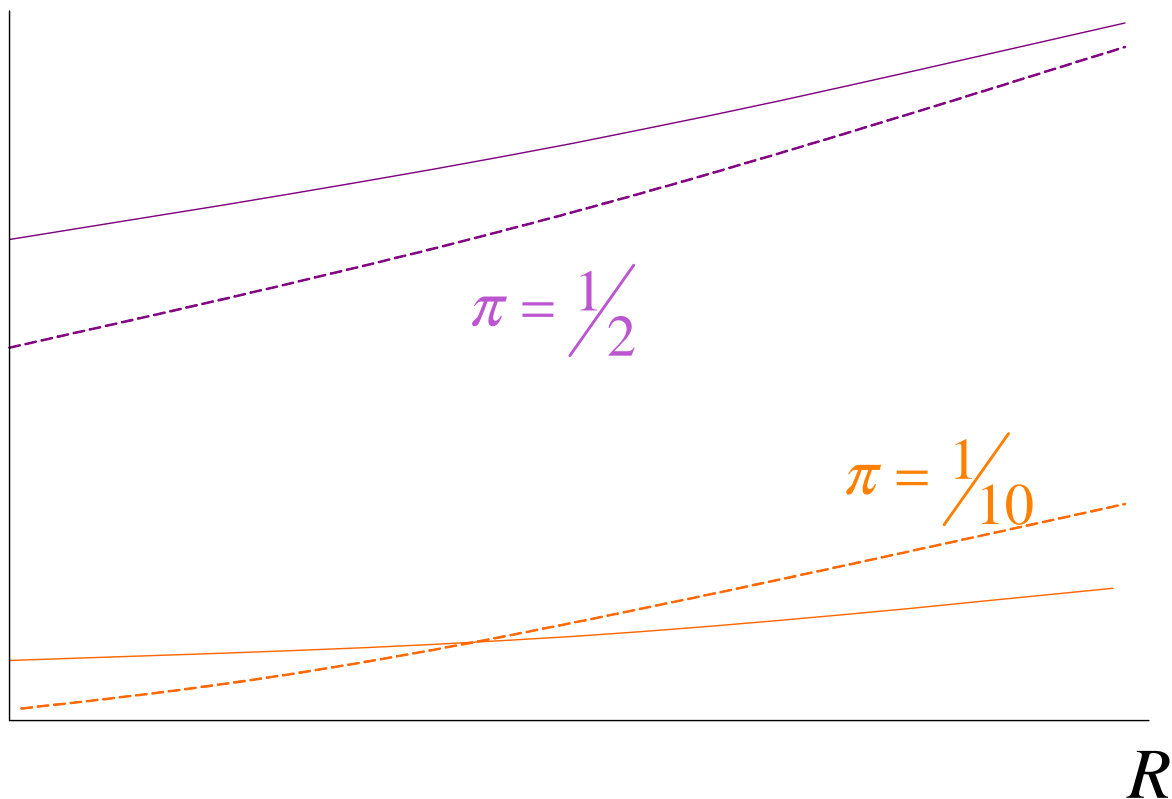
Proposition 8 *For $q + s + 2\frac{R\mathbb{E}[\theta]}{2-R} \geq 1$, the following equilibrium exists for the united executive with moral hazard and underlying types:*

- i. Voters use \hat{P} as their voting function*
- ii. Politicians expend $(e_a^*, e_b^*) = (\frac{R\mathbb{E}[\theta]}{2-R}, \frac{R\mathbb{E}[\theta]}{2-R})$*

Now the comparison of welfare across institutional structures is in favor of the united executive as long as $\mathbb{E}[\theta]$ is high enough. Note that the fact that a symmetric effort vector can be implemented is beneficial to the voters because none of the distortions of the previous subsection take place: in the first period voters obtain the maximum utility possible given the amount of total effort, and in the second period there is no informational externality across tasks which means that effort does not contaminate the ability to select. On the contrary, it can be shown that in this equilibrium, $\frac{\partial V_u(2)}{\partial R} > 0$, which implies that in this case effort actually helps selecting good types. The intuition for this is interesting: in this equilibrium, only politicians that provide two successes are reelected. Hence, the losses stem mainly from the good types that are ousted because they only happen to obtain a single success. Increasing effort has a linear return of order $2q$ for the competent types and only $2s$ for the incompetent. Hence, when effort increases good types are favored and selection is improved.

Figure 6 shows the welfare comparison between the divided government and the united when the balanced equilibrium exists. Two situations are depicted in which $s = \frac{1}{4}$ and $q = \frac{3}{4}$, for two different values of π . When π is small, $\mathbb{E}[\theta]$ is also small which reduces effort in the balanced equilibrium. The reason is that even though good types obtain successes with ease, there are very few of them in the population and agents assume that they are bad types. And clearly, if they are bad types there is not much return to putting effort because they need two successes to be reelected. Hence, when $\mathbb{E}[\theta]$ is small and R high enough, the divided government dominates.

Welfare



However, as shown when $\pi = \frac{1}{2}$, for higher $\mathbb{E}[\theta]$ there is no R for which divided government is preferable.

5.3 Discussion

Even though a balanced equilibrium is generally preferable from the point of view of the voters, the two types of united equilibria relate similarly to the equilibrium in a divided executive.²¹ In both case, for a divided executive to be preferable, we need both high R and small incidence of types (either small $\pi(1 - \pi)$, or small $q - s$). Hence the united executive dominance is robust to the inclusion of moderate moral hazard concerns.

This provides an answer to the question that we asked in section III. Given that it is so

²¹It can be shown that these equilibria coexist for some parameter values. In particular, the unbalanced equilibrium is available for all $R \leq \frac{1-q-s}{1-\mathbb{E}[\theta]}$ while the balanced equilibrium exists for $R \geq \frac{2(1-q-s)}{1+2\mathbb{E}[\theta]-q-s}$. Thus, if $\mathbb{E}[\theta] > \frac{1+q+s}{4}$ there is a region of multiple equilibria. In any case, when these equilibria coexist, voter's welfare is always higher in the balanced one.

difficult to obtain interior effort from a united executive, why do we observe an overwhelming presence of this institutional setting? The answer is that adding underlying types to the model provides a powerful rationale: at low levels of R , when effort is not important, a united executive dominates because of the technological advantage in the first period and the additional informational advantage in the second. As R increases both these advantages are eroded and for R high enough, a divided executive is preferable. This happens because in the unbalanced equilibrium voters cannot commit to the voting function that would extract a better allocation of effort. However, note that at relatively high levels of R , the balanced equilibrium becomes available. As long as types are important, this equilibrium dominates a divided executive because when voters expect high effort they can commit to reelect only in the case of two successes and there is no conflict between extracting effort and selecting types.

Hence, even in a simple model in which there are no considerations of scope across tasks, a united executive can be rationalized by the need to obtain more information about the type of the incumbent.

6 Conclusion

We have shown that introducing multitasking into political hidden action models has an adverse effect on the amount of effort that can be extracted from the politician if these different tasks are complementary in the preferences of the citizenry. In particular, the second-order conditions of the politician's problem put a binding constraint on the problem of the voters. Further, we have shown that this problem can be alleviated by dividing the government into two separately elected offices, each responsible for a separate dimension of outcomes. Thus, if elections were solely about incentivizing politicians to exert effort, then we should see separately elected ministers, instead of one executive being given many disparate responsibilities.

One possible explanation for this is that politicians are reelected on the basis of their type, not as a reward for their previous efforts. Staying in the context of political agency models, in the case of pure selection, it is always better to unite the functions of government under one executive, and hence the current institutional structure for chief executives makes sense. This finding is robust to including hidden action concerns to the selection problem. Indeed, when both

hidden information and hidden actions are present, then it is still very often the best choice to unite the functions of government under one executive, even if more effort can be extracted by dividing the functions of government. Hence, only political agency models which take into account underlying types can rationalize the pervasive existence of executives accountable for many outcome dimensions.

7 Appendix

Proof of Proposition 1. The first order conditions of program (3) are:

$$\begin{aligned} R[e_b(P_{GG} - P_{GB} - P_{BG} + P_{BB}) + P_{GB} - P_{BB}] &= e_a + e_b \\ R[e_a(P_{GG} - P_{GB} - P_{BG} + P_{BB}) + P_{BG} - P_{BB}] &= e_b + e_a \end{aligned}$$

The Hessian of this problem is:

$$\begin{pmatrix} -1 & R(P_{GG} - P_{GB} - P_{BG} + P_{BB}) - 1 \\ R(P_{GG} - P_{GB} - P_{BG} + P_{BB}) - 1 & -1 \end{pmatrix}$$

Which has the following determinant:

$$R(P_{GG} - P_{GB} - P_{BG} + P_{BB})[2 - R(P_{GG} - P_{GB} - P_{BG} + P_{BB})]$$

This determinant is positive only if $P_{GG} - P_{GB} - P_{BG} + P_{BB} \geq 0$.

Ignore for the moment restrictions $P_{GG} > 0$, $P_{BB} < 1$ and $0 \leq P_{GB}, P_{BG} \leq 1$. State program (5) as follows:

$$\begin{aligned} & \max_{(P_{GG}, P_{BB}, P_{BG}, P_{GB}, e_a, e_b) \in [0,1]^6} e_a \\ \text{sbj to} & \quad e_b \geq K \quad \alpha \\ & \quad P_{GG} \leq 1 \quad \lambda \\ & \quad P_{BB} \geq 0 \quad \mu \\ R(P_{GG} - P_{GB} - P_{BG} + P_{BB})[2 - R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] & \geq 0 \quad \beta \\ R[e_b(P_{GG} - P_{GB} - P_{BG} + P_{BB}) + P_{GB} - P_{BB}] - e_a - e_b &= 0 \quad \gamma \\ R[e_a(P_{GG} - P_{GB} - P_{BG} + P_{BB}) + P_{BG} - P_{BB}] - e_b - e_a &= 0 \quad \zeta \end{aligned}$$

The first order conditions of the lagrangian yield:

$$1 - \gamma + \zeta[R(P_{GG} - P_{GB} - P_{BG} + P_{BB}) - 1] = 0 \quad (8)$$

$$\alpha + \gamma[R(P_{GG} - P_{GB} - P_{BG} + P_{BB}) - 1] - \zeta = 0 \quad (9)$$

$$-\lambda + R\beta[2 - 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] + \gamma Re_b + \zeta Re_a = 0 \quad (10)$$

$$\mu + R\beta[2 - 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] + \gamma R(e_b - 1) + \zeta R(e_a - 1) = 0 \quad (11)$$

$$R\beta[-2 + 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] + \gamma R(1 - e_b) - \zeta Re_a = 0 \quad (12)$$

$$R\beta[-2 + 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] - \gamma Re_b + \zeta R(1 - e_a) = 0 \quad (13)$$

Substract (13) from (12) and get

$$\gamma R(1 - e_b) - \zeta Re_a + \gamma Re_b - \zeta R(1 - e_a) = 0$$

This expression implies that $\gamma = \zeta$. Using (8) one concludes: $\gamma = \zeta > 0$

Rewrite (12) and (13) as:

$$\beta[-2 + 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] + \gamma(1 - e_a - e_b) = 0 \quad (14)$$

Subtracting (9) from (8) and plugging in $\gamma = \zeta$ implies $\alpha = 1$. Hence we learn that the tangent at the optimum has slope -1 .

Rewrite (10) and (11) as:

$$-\lambda + R\beta[2 - 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] + \gamma R(e_b + e_a) = 0 \quad (15)$$

$$\mu + R\beta[2 - 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] + \gamma R(e_b + e_a - 2) = 0 \quad (16)$$

Now, take (14) as:

$$\beta[-2 + 2R(P_{GG} - P_{GB} - P_{BG} + P_{BB})] = \gamma(e_a + e_b - 1)$$

Substitute into (15) and obtain:

$$-\lambda - R\gamma(e_a + e_b - 1) + \gamma R(e_b + e_a) = 0$$

which implies $\lambda = \gamma R > 0$. Hence $P_{GG} = 1$.

Substitute (14) into (16) and get:

$$\mu - R\gamma(e_a + e_b - 1) + \gamma R(e_b + e_a - 2) = 0$$

which also implies $\mu = \gamma R > 0$. Hence $P_{BB} = 0$. This proves part *i.* of the proposition.

By (14), $\beta = 0$ (except when $e_b + e_a = 1$, see below) and part *ii.* is proven.

Hence, the solution to the program is determined by:

First Order Conditions of the agent:

$$e_b + e_a = \frac{R(P_{GB} + P_{BG})}{2 - R(1 - (P_{GB} + P_{BG}))} \quad (17)$$

Now, $\frac{\partial(e_b + e_a)}{\partial(P_{GB} + P_{BG})} > 0$ and hence the Concavity Constraint is binding. This implies that:

$$1 - P_{GB} - P_{BG} = 0 \quad (18)$$

Hence we obtain in the interior:

$$e_b + e_a = \frac{R}{2}$$

And the optimal contract is given by $P_{GG} = 1$, $P_{BB} = 0$ and $P_{GB} = P_{BG} = \frac{1}{2}$. This last result is true because $P_{GB} = P_{BG}$ is implied by the first order conditions of the problem of the agent with (18) plugged in. Note that these conclusions imply that $P_{GG} > 0$ and $P_{BB} < 1$. The concavity constraint is rewritten as $P_{GB} + P_{BG} = 1$, hence these two parameters are interior.

The restrictions ignored at the beginning are, thus, satisfied.

Finally, $e_b + e_a = 1$ together with (17) implies $R = 2$. Which is outside of the considered parameter space. ■

Proof to Proposition 3:

The problem of the agent is

$$\max_{e_a, e_b} e_a e_b w_2 + e_a(1 - e_b)w_1 + e_b(1 - e_a)w_1 - \frac{1}{2}(e_a + e_b)^2$$

which yields the first order conditions

$$e_b(w_2 - 2w_1) + w_1 = e_a + e_b$$

$$e_a(w_2 - 2w_1) + w_1 = e_a + e_b$$

The Hessian becomes

$$\begin{pmatrix} -1 & w_2 - 2w_1 - 1 \\ w_2 - 2w_1 - 1 & -1 \end{pmatrix}$$

with the determinant:

$$2(w_2 - 2w_1) - (w_2 - 2w_1)^2$$

which implies the Concavity constraint:

$$w_2 - 2w_1 > 0$$

Hence, the problem of the principal can be stated as:

$$\max_{e_a, e_b, w_1, w_2} e_a e_b(1 - w_2) + e_a(1 - e_b)(\zeta - w_1) + e_b(1 - e_a)(\zeta - w_1)$$

$$\text{sbj. to: } e_b(w_2 - 2w_1) + w_1 = e_a + e_b \quad \lambda$$

$$e_a(w_2 - 2w_1) + w_1 = e_a + e_b \quad \mu$$

$$w_2 - 2w_1 > 0 \quad \gamma$$

Which yields the following first order conditions:

$$e_b(1 - w_2) + (1 - e_b)(\zeta - w_1) - e_b(\zeta - w_1) - \lambda + \mu(w_2 - 2w_1 - 1) = 0 \quad (19)$$

$$\begin{aligned} e_a(1 - w_2) + (1 - e_a)(\zeta - w_1) - e_a(\zeta - w_1) - \mu + \lambda(w_2 - 2w_1 - 1) &= 0 \\ -e_a e_b + \lambda e_b + \mu e_a + \gamma &= 0 \end{aligned} \quad (20)$$

$$-e_a(1 - e_b) - e_b(1 - e_a) + \lambda(1 - 2e_b) + \mu(1 - 2e) - 2\gamma = 0 \quad (21)$$

Let's find the symmetric point: $e_a = e_b \equiv e$, $\lambda = \mu$.

Take (20):

$$-e^2 + 2\lambda e + \gamma = 0$$

And Take (21):

$$-2e(1 - e) + 2\lambda(1 - 2e) - 2\gamma = 0$$

Now let's examine case by case:

Assume $\gamma = 0$ and $e > 0$. Now (20) implies $e = 2\lambda$. But then (21) implies $e = 0$, a contradiction.

Assume $\lambda = 0$ and $e > 0$. Now (20) implies $\gamma = e^2$. But then (21) implies $e = 0$, a contradiction.

Hence it has to be that $\lambda > 0$ and $\gamma > 0$. This proves the proposition.

We characterize the solution. The first order condition is binding:

$$e = \frac{w_1}{2 - w_2 + 2w_1}$$

and the concavity constraint is binding $w_2 - 2w_1 = 0$. Which implies:

$$e = \frac{w_1}{2}$$

Finally, take (20) and (21) and it is easy to show that $\lambda = e$. And now we can take (19) and substitute in:

$$w_1 = \frac{\zeta}{\zeta + \frac{3}{2}}$$

Which equals total effort in the interior. This total effort is higher than in the focused allocation if $\zeta < \frac{1}{2}$.

Proof to Lemma 3:

See the proof to Lemma 4 for the particular case $e_a^* = e_b^* = 0$.

Proof to Proposition 4:

Assume first that $s + q \leq 1$. Hence Lemma 3 implies that one good signal is enough to reelect.

First define voters' welfare as

$$\bar{V}(p_a, p_b) = p_a p_b + p_a (1 - p_b) \zeta + p_b (1 - p_a) \zeta$$

where p_j takes on the values of q and s . Hence,

$$\begin{aligned} V_u(1) &= \pi \bar{V}(q, q) + (1 - \pi) \bar{V}(s, s) \\ &\quad \left(\pi (1 - q)^2 + (1 - \pi) (1 - s)^2 \right) (\pi V(q, q) + (1 - \pi) V(s, s)) + \\ V_u(2) &= \pi (q^2 + 2q(1 - q)) \bar{V}(q, q) + \\ &\quad (1 - \pi) (s^2 + 2s(1 - s)) \bar{V}(s, s) \end{aligned}$$

For a divided government,

$$\begin{aligned} &\pi^2 \bar{V}(q, q) + \\ V_d(1) &= 2\pi (1 - \pi) \bar{V}(q, s) + \\ &\quad (1 - \pi)^2 \bar{V}(s, s) \\ &\quad \left(\pi^2 (1 - q)^2 + 2\pi (1 - \pi) (1 - q) (1 - s) + (1 - \pi)^2 (1 - s)^2 \right) V_d(1) \\ &\quad \left(2\pi (1 - \pi) q (1 - s) + \pi^2 q (1 - q) \right) (\pi \bar{V}(q, q) + (1 - \pi) \bar{V}(q, s)) \\ V_d(2) &= \left((1 - \pi)^2 s (1 - s) + 2\pi (1 - \pi) s (1 - q) \right) (\pi \bar{V}(q, s) + (1 - \pi) \bar{V}(s, s)) \\ &\quad 2\pi (1 - \pi) s q \bar{V}(q, s) + \\ &\quad (1 - \pi)^2 s^2 \bar{V}(s, s) + \\ &\quad \pi^2 q^2 \bar{V}(q, q) \end{aligned}$$

It is easy to see that

$$V_u(1) - V_d(1) = (1 - 2\zeta) (q - s)^2 \pi (1 - \pi)$$

and so the voters are better off in the first period with united government as long as $\zeta \leq \frac{1}{2}$.

We now calculate the second period utilities as

$$V_u(2) - V_d(2) = (q - s)^2 \pi (1 - \pi) (A(q, s, \pi) + \zeta B(q, s, \pi))$$

where $A(q, s, \pi)$ and $B(q, s, \pi)$ are known functions of q, s , and t . It can be shown that, for all $0 \leq \pi \leq 1$, $0 \leq s \leq q \leq 1$ and $s + q \leq 1$, $B(q, \pi, t) \geq 0$. Hence this function is always increasing in ζ . Since for the same domain $A(q, s, \pi) + \frac{1}{2}B(q, s, \pi) \geq 0$, we conclude $V_u(2) - V_d(2) \geq 0$.

Assume now that $s + q \geq 1$. Now it is optimal to reelect only when two positive outcomes are observed. In this case, we have

$$\begin{aligned} V_u(1) &= \pi \bar{V}(q, q) + (1 - \pi) \bar{V}(s, s) \\ &\quad (\pi(1 - q^2) + (1 - \pi)(1 - s^2)) (\pi V(q, q) + (1 - \pi) V(s, s)) + \\ V_u(2) &= \pi q^2 \bar{V}(q, q) + \\ &\quad (1 - \pi) s^2 \bar{V}(s, s) \end{aligned}$$

The first period utilities remain the same, so

$$V_u(1) - V_d(1) = (1 - 2\zeta) (q - s)^2 \pi (1 - \pi)$$

and so the agents are always better off in the first period with united government.

We now calculate the second period utilities as

$$V_u(2) - V_d(2) = (q - s)^2 \pi (1 - \pi) (A'(q, s, \pi) + k B'(q, s, \pi))$$

where $A'(q, s, \pi)$ and $B'(q, s, \pi)$ are known functions of q, s , and t . Again, it can be shown for all $0 \leq \pi \leq 1$, $0 \leq s \leq q \leq 1$ and $s + q \leq 1$ that $B'(q, s, \pi) \geq 0$. Further, $A'(q, s, \pi) + \frac{1}{2}B'(q, s, \pi) \geq 0$ on that domain as well. Hence, agents are weakly better off in the second period under united government as well.

Proof to Proposition 5:

Under divided government, the agent wishes to solve

$$\max_{e_i} \left\{ \frac{R}{2} \mathbb{E}_\theta [P_G (e_i + \theta) + P_B (1 - (e_i + \theta))] - \frac{1}{2} e_i^2 \right\}$$

The first order condition for the agent gives us that

$$\tilde{e}_i = \frac{R}{2} (P_G - P_B)$$

For the voters, simple use of Bayes' Rule yields:

$$\begin{aligned} \mu(G) &= \frac{\pi(q + \tilde{e})}{\pi(q + \tilde{e}) + (1 - \pi)(s + \tilde{e})} > \pi \\ \mu(B) &= \frac{\pi(1 - (q + \tilde{e}))}{\pi(1 - (q + \tilde{e})) + (1 - \pi)(1 - (s + \tilde{e}))} < \pi \end{aligned}$$

so they will choose

$$P_G = 1, P_B = 0$$

and so

$$e_i^d = \frac{R}{2}$$

Proof to Lemma 4:

Given an equilibrium level of effort $(\tilde{e}_a, \tilde{e}_b)$, the voters will update their beliefs on the type of the politician. We have that

$$\mu(GG|\tilde{e}_a, \tilde{e}_b) = \frac{\pi(q + \tilde{e}_a)(q + \tilde{e}_b)}{\pi(q + \tilde{e}_a)(q + \tilde{e}_b) + (1 - \pi)(s + \tilde{e}_a)(s + \tilde{e}_b)} > t$$

and

$$\mu(BB|\tilde{e}_a, \tilde{e}_b) = \frac{\pi(1 - (q + \tilde{e}_a))(1 - (q + \tilde{e}_b))}{\pi(1 - (q + \tilde{e}_a))(1 - (q + \tilde{e}_b)) + (1 - \pi)(1 - (s + \tilde{e}_a))(1 - (s + \tilde{e}_b))} < t$$

so the principal, acting optimally, must set

$$P_{GG} = 1, P_{BB} = 0$$

Now, we also have

$$\begin{aligned}\mu(GB|\tilde{e}_a, \tilde{e}_b) &= \frac{\pi(q + \tilde{e}_a)(1 - (q + \tilde{e}_b))}{\pi(q + \tilde{e}_a)(1 - (q + \tilde{e}_b)) + (1 - \pi)(s + \tilde{e}_a)(1 - (s + \tilde{e}_b))} \\ \mu(BG|\tilde{e}_a, \tilde{e}_b) &= \frac{\pi(1 - (q + \tilde{e}_a))(q + \tilde{e}_b)}{\pi(1 - (q + \tilde{e}_a))(q + \tilde{e}_b) + (1 - \pi)(1 - (s + \tilde{e}_a))(s + \tilde{e}_b)}\end{aligned}$$

and these are greater than π whenever $\tilde{e}_a + \tilde{e}_b + s + q < 1$. To see this note that

$$\begin{aligned}\mu(GB|\tilde{e}_a, \tilde{e}_b) > \pi &\Leftrightarrow \frac{\pi}{\pi + (1 - \pi)\frac{(s + \tilde{e}_a)(1 - (s + \tilde{e}_b))}{(q + \tilde{e}_a)(1 - (q + \tilde{e}_b))}} > \pi \\ &\Leftrightarrow (q + \tilde{e}_a)(1 - (q + \tilde{e}_b)) > (s + \tilde{e}_a)(1 - (s + \tilde{e}_b)) \\ 0 < (q - s)(1 - (q + s + \tilde{e}_a + \tilde{e}_b))\end{aligned}$$

and since $q - s > 0$ by assumption, we have that

$$\mu(GB|\tilde{e}_a, \tilde{e}_b) > \pi \Leftrightarrow 1 > q + s + \tilde{e}_a + \tilde{e}_b$$

An equivalent calculation can be done for $\mu(BG|\tilde{e}_a, \tilde{e}_b)$.

Proof to Proposition 6:

Assume that $q + s + e_a^* + e_b^* < 1$. In this case Lemma 4 implies that \tilde{P} is the only strategy that the voters can play. In view of this, the problem of the agent becomes:

$$\max_{e_a, e_b} \left\{ R[-e_a e_b + e_a(1 - \mathbb{E}[\theta]) + e_b(1 - \mathbb{E}[\theta])] - \frac{1}{2}(e_a + e_b)^2 \right\}$$

The first order conditions of this program are:

$$R(1 - e_b - \mathbb{E}[\theta]) = e_a + e_b$$

$$R(1 - e_a - \mathbb{E}[\theta]) = e_a + e_b$$

However, the Hessian of the program is negative, hence it is not globally concave and the solution must be in a corner. It is then clear that:

$$e_i^* = R(1 - \mathbb{E}[\theta]), e_j^* = 0$$

are the two solutions.

For \tilde{P} to be optimal, we thus need $q + s + R(1 - \mathbb{E}[\theta]) < 1$

Proof to Proposition 7:

We state the different welfare components:

$$V_u(1) \equiv \pi q (q + e^*) + (1 - \pi) s (s + e^*)$$

From which it is immediate:

$$\frac{\partial V_u(1)}{\partial R} = \frac{\partial V_u(1)}{\partial e^*} \frac{\partial e^*}{\partial R} = \mathbb{E}[\theta](1 - \mathbb{E}[\theta]) > 0$$

In the second period:

$$V_u(2) \equiv \begin{cases} \pi (1 - (q + e^*)) (1 - q) q^2 + \\ (1 - \pi) (1 - (s + e^*)) (1 - s) s^2 + \\ (1 - \pi (1 - (q + e^*)) (1 - q) - (1 - \pi) (1 - (s + e^*)) (1 - s)) (\pi q^2 + (1 - \pi) s^2) \end{cases}$$

Which can be simplified to:

$$V_u(2) = K(q, s, \pi) + (q - s)^2 (q + s) (1 - \pi) \pi (\mathbb{E}[\theta] - 1) R$$

From which it is immediate that $\frac{\partial V_u(1)}{\partial R} < 0$.

Now we go to the expressions for divided executive:

$$V_d(1) \equiv \begin{cases} \pi^2 (q + e_i^d)^2 + \\ 2\pi (1 - \pi) (q + e_i^d) (s + e_i^d) + \\ (1 - \pi)^2 (s + e_i^d)^2 \end{cases}$$

From which we obtain:

$$\frac{\partial V_d(1)}{\partial R} = \frac{R}{2} + \mathbb{E}[\theta] > 0$$

And clearly $\frac{\partial V_d(1)}{\partial R} > \frac{\partial V_u(1)}{\partial R}$.

Finally, we have

$$V_d(2) \equiv \left\{ \begin{array}{l} \pi^2 \left((q + e_i^d)^2 q^2 + 2(q + e_i^d)(1 - (q + e_i^d)) q \mathbb{E}[\theta] + (1 - (q + e_i^d))^2 \mathbb{E}[\theta^2] \right) + \\ 2\pi(1 - \pi) \left(\begin{array}{l} (q + e_i^d)(s + e_i^d) qs + (q + e_i^d)(1 - (s + e_i^d)) q \mathbb{E}[\theta] + \\ (1 - (q + e_i^d))(s + e_i^d) \mathbb{E}[\theta] s + (1 - (q + e_i^d))(1 - (s + e_i^d)) \mathbb{E}[\theta^2] \end{array} \right) + \\ (1 - \pi)^2 \left((s + e_i^d)^2 s^2 + 2(s + e_i^d)(1 - (s + e_i^d)) s \mathbb{E}[\theta] + (1 - (s + e_i^d))^2 \mathbb{E}[\theta^2] \right) \end{array} \right.$$

Plugging in e_i^d , the expression simplifies to

$$V_d(2) = (s^2(1 - \pi)\pi + q + q^2(1 - \pi) - s(\pi - 1)(2q\pi - 1))^2$$

Which is independent of R .

References

- [1] ALESINA, A. AND TABELLINI, G. 2003. "Bureaucrats or Politicians?" Manuscript, Harvard University.
- [2] ASHWORTH, S. 2005. "Reputational Dynamics and Political Careers." *Journal of Law, Economics and Organization*, 21: 441-466.
- [3] AUSTEN-SMITH, D., AND BANKS, J. 1989. "Electoral Accountability and Incumbency." in Ordeshook, ed. *Models of Strategic Choices in Politics*. University of Michigan Press.
- [4] BAKER, G. 1992. "Incentive Contracts and Performance Measurement." *Journal of Political Economy*, 100: 598-614.
- [5] BAKER, G. 2003. "Distortion and Risk in Optimal Incentive Contracts." *Journal of Human Resources*, 37: 728-751.
- [6] BANKS, J., AND SUNDARAM, R. 1993. "Moral Hazard and Adverse Selection in a Model of Repeated Elections." in Barnett et al., eds. *Political Economy: Institutions, Information, Competition and Representation*. Cambridge University Press.
- [7] BANKS, J., AND SUNDARAM, R. 1998. "Optimal Retention in Agency Problems." *Journal of Economic Theory*, 82: 293-323.
- [8] BARRO, R. 1973. "The control of politicians: an economic model." *Public Choice*, 14: 19-42.
- [9] BESLEY, T. 2006. *Principled Agents: Motivation and Incentives in Politics*. Oxford University Press.
- [10] BESLEY, T. AND CASE, A. 1995. "Incumbent Behavior, Vote Seeking, Tax Setting and Yardstick Competition." *American Economic Review*, 85: 25-45.
- [11] DEWATRIPONT, M., JEWITT, I. AND TIROLE, J. 1999. "The Economics of Career Concerns, Part II: Application to Missions and Accountability of Government Agencies." *The Review of Economics Studies*, 66: 199-217.
- [12] DIERMEIER, D., KEANE, M. AND MERLO, A. 2005. "A Political Economy Model of Congressional Careers." *American Economic Review*, 95: 347-373.

- [13] DIXIT, A. 1996. *The Making of Economic Policy: A Transaction Costs Politics Perspective*. MIT Press.
- [14] FEARON, J. 1999. "Electoral Accountability and the Control of Politicians: Selecting Good Types versus Sanctioning Poor Performance." in Przeworski et al., eds. *Democracy, Accountability and Representation*. Cambridge University Press.
- [15] FERREJOHN, J. 1986. "Incumbent Performance and Electoral Control." *Public Choice*, 50: 5-26.
- [16] GROSECLOSE, T. AND MILYO, J. 1999. "Buying the Bums Out: What's the Dollar Value of a Seat in Congress?" Manuscript, Stanford Graduate School of Business.
- [17] HOLMSTROM, B. 1979. "Moral Hazard and Observability" *The Bell Journal of Economics*, 10: 74-91.
- [18] HOLMSTROM, B., AND MILGROM, P. 1991. "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership and Job Design." *Journal of Law, Economics and Organization*, 7: 24-52.
- [19] LAFFONT, J-J. AND MARTIMORT, D. 2002. *The Theory of Incentives: The Principal-Agent Model*. Princeton University Press.
- [20] ROGOFF, K. 1990. "Equilibrium Political Budget Cycles." *The American Economic Review*, 80: 21-36.
- [21] ROGOFF, K. AND SIBERT, A. "Elections and Macroeconomic Policy Cycles." *The Review of Economic Studies*, 55: 1-16.
- [22] PERSSON, T., AND TABELLINI, G. 2000. *Political Economics: Explaining Economic Policy*. MIT Press.
- [23] SMART, M. AND STURM, D. 2004. "Term Limits and Electoral Accountability." Manuscript, University of Toronto.
- [24] SNYDER, J. AND TING, M. 2006. "Interest Groups and The Electoral Control of Politicians." Manuscript, Columbia University.

- [25] TING, M. 2002. "A Theory of Jurisdictional Assignments in Bureaucracies." *American Journal of Political Science*, 46: 364-378.