Strategic Risk, Civil War and Intervention*

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Abstract

This paper presents a theory of conflict in which violence occurs as a result of strategic risk. Actors face a difficult balancing act between the fear of being attacked and the opportunity cost of breaking peace that selects the risk dominant equilibrium. We link the propensity of conflict to current and future economic conditions and discuss the effects of growth, inequality and military technology on the ability of groups to escape the Security Dilemma.

Keywords : Conflict, Security Dilemma, Coordination failure, Global Games, Exit Games

PRELIMINARY – COMMENTS WELCOME

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1 Introduction

In weak states the government does not have a monopoly on the coercive use of violence. As a consequence, groups always have the option of imposing favourable settlements with the use of force. The technology of violence in intercommunal conflicts typically involves raids with militias on the civilian populations of the neighboring groups. Such technology provides a big first strike advantage because the effects of ethnic cleansing are very difficult to reverse. The characteristics of this technology thus typically lock neighboring groups in a coordination problem: they want to reap the proceeds of peace but want to avoid being attacked. As a consequence, groups with fundamentally compatible goals, may end up locked in violent conflict because of the mutual fear of an attack.

In the present paper, we propose a simple model of coordination in the presence of strategic risk. With this framework we want to explore the two-way relationship between economic variables and the risk of conflict to try to gain insight on the circumstances in which the inherent insecure peace in weak states will break down in conflict generated by mistrust. Also, we want to explore economic decisions under the shadow of conflict. Finally, we derive some general guidelines to determine when and where interventions such as foreign aid and peace-keeping are most useful.

In our model, two neighboring groups try to coexist in a territory. They take a single simultaneous decision: whether to keep peace, or to attack the other group. The offensive advantage is such that the best response to an expected attack is always to attack. However, by attacking the group foregoes the economic returns from peace. The proceeds of peace depend on two factors: the inherent wealth of the group and a continuous random shock, θ, that captures the current state of the economy. Players observe θ prior to taking their decision. In a poor agricultural economy, inherent wealth might be interpreted as land quality, and the shock can capture such variable economic conditions as rain. We show that in such a model, there is a level in the economic conditions low enough that war is inevitable as there are not enough resources to ensure the peaceful survival of both groups. Likewise, there are states of the world in which returns to peace are so abundant that nobody wishes to attack. The interesting question remains for average levels of the state of the economy. Will groups be able to coordinate into peace?

For these average realizations of the shock, both peace and violence are equilibria *under common knowledge of the state of the economy*. That is, when both groups know exactly the state of the world. Both know that both know it, and so on. Common knowledge permits, in equilibrium, to associate to any interval of these average realizations an equilibrium outcome of "peace" or "violence" as both of them are self-enforcing. However, the assumption of common knowledge in this setting completely erases the existence of strategic risk which is at the heart of the coordination problem. In other words, when a player chooses what to do, she knows exactly what the equilibrium prescribes the other player to do. There is no second-guessing and no fear of deviations as they never happen in equilibrium. To examine the effect of strategic risk in the capacity of these groups to coordinate we need to depart from common knowledge.

We depart by assuming that groups cannot perfectly see the realization of the state of the world. Instead, they observe a signal $\theta + \sigma\epsilon$, where ϵ is independently distributed across players and σ is a scalar. When σ goes to 0 groups know that they are observing almost the same signal. But this is not as good as common knowledge. Players are forced to second-guess the intentions of their opponent as they cannot anymore assume that they are observing exactly the same signal. This small uncertainty does not matter for very high, or very low levels of θ, as it is clear for the players what they should play. But in intermediate levels, miscoordination becomes a real possibility. As shown in Carlsson and van Damme (1993), this noise structure eliminates multiplicity and selects the risk-dominant equilibrium. In our model, this means that a single threshold equilibrium exists: when the realization of the state of the economy is below the threshold, the fear of miscoordiation is too big with respect to the returns to peace and groups fight. This threshold is well above the level of rain that allows for the peaceful equilibrium under common knowledge and hence we have violence when peace was possible. This violence is thus generated entirely by strategic risk and captures the intuition behind the Security Dilemma.

The comparative statics of this equilibrium provide interesting insights on the forces that exacerbate strategic fear. First, rich countries see less violence. This is because it is common knowledge that both groups have more to lose if they go to war. Hence, not only each one of them has smaller incentives to deviate, but they also know that their opponent has fewer incentives to attack and hence peace is reinforced. Second, the model exhibits deterrence in the sense the worse payoffs in the case of war support more peaceful equilibria. Third, quite naturally, higher payoffs in a first strike and worse payoffs in suffering such a first strike worsen coordination. The fear of miscoordinating increases as both groups know that there are higher incentives to deviate and worse payoffs in case of unilateral compliance. Intriguingly, inequality also exacerbates the coordination problem. This is true even though in our framework we make the payoff of looting independent of income.[1] It is worth noting

[1]Quite naturally, if proceeds from looting are increasing in the income of the victim, inequality will

that such natural comparative statics do not exist in the Pareto Optimal equilibrium under common knowledge.

We also examine a dynamic version of the game to explore whether forward looking groups can use future payoffs to help current coordination. Fearon and Laitin (1996) already emphasize that such dynamic considerations may be essential to explain why constant conflict is not prevalent. We model agents as playing an a exit game, in which violence causes the game to end. This simplification of the standard repeated game framework allows us to study questions related to economic growth and investment.

The first implication to be drawn from this analysis is that the is a static link between income and peace extends to a dynamic setting: the risk-dominance threshold is decreasing both in the current income level and in the future expected returns from the economy. As a consequence the common knowledge expectation of fast growth -in the absence of conflict- is conducive to stability. More generally, any future shock that increases the future proceeds of peace reduces the current probability of war. Obviously, the opposite occurs with future shocks that are expected to reduce economic returns.

Second, future economic inequality also affects the probability of war. The more unbalanced the proceeds of peace are, the lower are the returns to peace for the poor and thus the higher is the probability of violence. In addition, future inequality has a compounding effect because the increased probability of future violence reduces the option value of peace thus increasing the incentives to deviate in the current period. An important implication of this analysis is that economic growth is not enough to prevent outbreaks of violence. The proceeds of economic development have to be reasonably shared among the different groups to prevent inequality from fueling conflict.

Third, the model can generate a war trap in which poor countries cannot escape their plight because the high probability of conflict depresses investment, thus keeping the country poor and the probability of conflict high. By themselves, even if there is a string of good economic shocks, these countries cannot escape their poverty-violence vicious cycle. On the other hand, if a country reaches a certain level of income per capita, a virtuous cycle can set off in which investment and future growth reduce the current probability of conflict and hence countries can grow out of violence.

With these results, the paper provides a framework to address policy questions such as the form and optimal duration of a peace-keeping intervention operation. The analysis

increase the probability of conflict. We consider this a "predatory" motive for war and hence we abstract from this effect.

shows that when there is economic growth during peace periods the probability of reversion to violence is decreasing in the duration of the peace. Hence, keeping a force that prevents violence for some initial period, may have dramatic long term effects on the stabilization of a country: the economy can be allowed to grow up to a point in which the incentives to keep peace make coordination easy. To our knowledge, this is the first model that rationalizes temporary peace-keeping interventions. In addition, we also derive some policy implications on the optimal use of economic aid to groups or countries to minimize the occurrence of violence.

This paper is related to various streams of literature. Starting with Herz (1950) and structured in Jervis (1976, 1978) the concept of the Security Dilemma is used as the staple theory of realist security studies scholars to analyze the causes of escalation and ultimately of war.[2] At the heart of this concept is the acknowledgement of a state of anarchy in international relations which makes commitment difficult. In such circumstances, whenever a country sees a neighbor making military preparations, it can deduce that it intends to attack (as opposed to prepare for defense) and respond by escalating its own preparations. Realist scholars argue that such process leads to a spiral that generates actual conflict. While we do not present a model of the Security Dilemma, we microfound the existence of strategic uncertainty which is necessary for this argument to have any relevance and we relate it to the economic environmet in which this risk resides. Kydd (1997) and Baliga et al. (2004) provide formal interpretations of the Security Dilemma in models based on the existence of aggressive types. In these models, contenders may end up locked in conflict because there is some probability that the opponent is predatory, that is, it may want to attack even if it is given security guarantees. Our approach hinges on observational uncertainty about the state of the world where agents are equal and receive almost identical information. In a sense, type based approaches study how the possibility of the opponent being truly aggressive affects conflict, while our approach emphasizes the effects that the economic environment has on the contenders' ability to coordinate into peaceful coexistence. We view the two approaches as highly complementary.

Posen (1993) pioneered the application of the Security Dilemma concept to a situation of ethnic confrontation.[3] In his analysis, the collapse of such states as the Soviet Union and Yugoslavia left the different ethnic entities living within their borders in a situation tantamount to the traditional realist anarchy. In general, this idea is applicable to any

[2]Glaser (1997) describes the concept as "the key to understanding how in an anarchical international system states with fundamentally compatible goals still end un in competition and war."

[3]See also Jervis and Snyder (1999) and Roe (2005)

circumstance in which there is no strong overarching authority, such as in sub-Saharan Africa where states are weak, or in international relations where there is no strong extra-governmental peace enforcer. The countervailing force that we emphasize is the opportunity cost of conflict. The fact that agents generally manage to cooperate when the costs are high enough, has already been recognized in the literature on ethnic conflict by Fearon and Laitin (1996), and in the literature on international organization by Glaser (1994).

Section 2 presents the static version of the game and discusses war as a coordination problem. Section 3 analyzes the dynamic model with exogenous growth dynamics. Section 4 analyzes the nature of the War Trap that can arise when we endogenize investment decisions. Section 5 draws policy implications and section 6 concludes. All proofs are contained in the appendix unless noted otherwise.

2 Conflict as Miscoordination

2.1 The Model

We consider a game played between two coexisting groups, A and B, that decide whether to remain peaceful P or go to war W. The payoff matrix of row player $i \in \{A, B\}$ is:

	P	W
P	$\Pi_i + \theta$	$\theta - S$
W	M	$-W$

These payoffs capture the following paradigmatic situation in an ethnically diverse, poor country. When a group decides to play peace P, it devotes its effort to some productive activity. In the absence of an attack, this productive activity yields a return that depends on the amount of productive capital of the group – denoted $\Pi_i \geq 0$ – and on the state of the world θ. The state of the economy θ is common to both groups.[4] It is distributed according to function F_θ, with continuous density f_θ and support in $\mathbb{R}$. If a group suffers an undefended attack, it loses the returns to productive capital. In addition, it suffers a loss of utility S that accounts for the violence it has to suffer. When a group attacks, we assume it completely abandons any productive activity. If it faces no resistance (if the opponent plays P) it obtains a payoff M. Finally, when both players attack, no production is made and

[4] In fact we only need the state of the world to be common at the stage where groups make decisions. Idiosyncratic noise could be added to the realized returns.

both groups incur a cost W for the violence they inflict on each other.[5]

With this payoff matrix we capture the strategic situation of groups in poor countries that can devote their efforts either to peaceful economic activities or to pillaging on their neighbours. For example, in a rural setting this economic activity is agriculture and $\Pi_i + \theta$ are the returns that depend on capital such as land quality or irrigation and a random shock such as rainfall. An alternative example is the coexistence of agricultural and pastoral groups that need to share water and land but also engage in cattle rustling and violently preclude each other's access to vital water sources and grazing land. This situation is typical of countries in the Sahel and the Horn of Africa. Some analysts and journalistic accounts put these tensions at the basis of the violence in Darfur and Sudan and, at a lower level, the communal clashes in Ethiopia and northern Kenya.

To simplify notation and without loss of generality, we assume $\Pi_A \geq \Pi_B$. To avoid equilibria in which groups passively accept attacks by the opponents, we impose a restriction on payoffs. More precisely, we assume that,

$$S - W > M - \Pi_B \tag{1}$$

This ensures that the best response to violence when the opponent finds it profitable attack, is also to attack.

This payoff matrix can potentially incorporate the two elements of decentralized violence that we want to emphasize: the fear of being attacked as a reason to go to war and the economic opportunity cost as a reason to keep the peace.[6] However, with common knowledge of the state of the economy θ, the strategic fear of being attacked is not really present. To see this, note that when θ is common knowledge, this game falls into one of three regimes:

1. When $\theta < \underline{\theta} \equiv M - \Pi_B$, the game features a single Nash equilibrium at (W, W).[7]

[5] This payoff matrix is formulated in very simple terms to simplify the calculations below. However, the insights provided are very general. In particular, all we need is that the difference between playing P and playing W is increasing in θ for any action of the opponent.

[6] This characterization of the violence dilemma is already present in Hobbes' Leviathan:

> "others may probably be expected to come prepared with forces united to dispossess and deprive him not only of the fruit of his labor, but also of his life or liberty. [...] And from this diffidence of one another, there is no way for any man to secure himself so reasonable as anticipation, that is, by force or wiles, to master the persons of all men he can" Later in the same chapter, he writes "The passions that incline men to peace are fear of death, desire of such things as are necessary to commodious living, and a hope by their industry to obtain them."

[7] The game is actually a Prisoners' dilemma only if $\theta < M - \Pi_A$.

2. When $\underline{\theta} \leq \theta \leq \bar{\theta}$ there are multiple equilibria at (P, P) and (W, W)

3. When $\theta > \bar{\theta} \equiv S - W$ a unique Nash equilibrium exists at (P, P)

In the first region, the return to peace is too small to avoid violence and the game can be solved by iterated dominance. This situation captures cases in which drought is so severe that the peaceful sharing rule does not guarantee the survival of the poor group. On the contrary, in the third region returns to peace are so high that nobody expects anybody to attack. For θ in this region the game can also be solved by iterated elimination.

In practical terms, these extreme states of the world occur with very small probability so even though our results hold in full generality, we are thinking of settings in which $F_\theta(\underline{\theta})$ and $1 - F_\theta(\bar{\theta})$ are very small.

For intermediate states of the world, this payoff matrix involves a non-trivial coordination problem in which both peace and war are sustainable as equilibrium outcomes. Hence, the central region is the area of interest in which war can arise because of a coordination problem. However, with common knowledge of θ this coordination problem does not capture strategic risk. There are infinitely many equilibria as any mapping from any interval of $\theta \in [\underline{\theta}, \bar{\theta}]$ into (P, P) or (W, W) can be sustained in equilibrium. However, note that in each one of these equilibria, each player knows exactly what the opponent will do.[8] With common knowledge and perfect information, equilibrium does not entail any "second-guessing" the intentions of the opponent. Hence, this intuitive element of strategic risk is not present.

To introduce strategic uncertainty, we depart from common knowledge by imposing an assumption that is very realistic in the contexts we are interested in. In developing economies, economic returns are very closely correlated due to the reliance on weather. However it can be quite difficult to predict with precision and to agree on the state of the world that will occur. Hence we assume that players observe a signal $s = \theta + \sigma\varepsilon_i$ where θ is the state of the world and ε_i, $i = A, B$ is idiosyncratic observational noise that is independently distributed. We denote by Γ_σ the resulting game with incomplete information.

With this information structure we follow Carlsson and van Damme (1993) which show that if players have precise but still imperfect information about the state of the world, the set of rationalizable strategies is a singleton that converges to the risk-dominant equilibrium as σ goes to 0.

[8] The existence of multiple equilibria means that many outcomes are possible. However, once players know which equilibrium they are playing, there is no strategic risk left. We can envisage groups as observing θ and then referring to the equilibrium strategies to find out what they are supposed to play. In other words, in equilibrium, strategic risk does not cause violence.

Lemma 1. *As σ goes to 0, the set of rationalizable strategies of game Γ_σ converges to a singleton (x_A, x_B). Moreover, strategies x_A and x_B are the risk dominant strategies: players cooperate if and only if θ is greater than the risk dominant threshold θ^{RD}.*

The application of global games to our context is particularly appealing: when a group looks at the sky and predicts whether rain will be forthcoming or not it cannot be sure of how many clouds the neighboring group is noticing. This makes the risk of miscoordinating real. In such circumstances, the group will balance the returns to peace with its assessment of the propensity of the opponent to attack. By breaking common knowledge in this way, we gain two things. First, multiplicity of equilibria is broken in favor of the risk-dominant equilibrium.[9] Second, and more important, in the risk-dominant equilibrium, strategic risk considerations are factored in because players never know exactly what the other player is observing.

2.2 Risk Dominance and Conflict

The risk dominant equilibrium corresponds to the one for which the product of unilateral deviations is largest.[10] Hence, in the context of our game, the risk dominant threshold is defined by the equation

$$(\Pi_A + \theta - M)(\Pi_B + \theta - M) = (-W - \theta + S)^2 \tag{2}$$

The left hand side (LHS) of this equation is the product of the unilateral deviation gain from the peaceful equilibrium for both players, while the right hand side (RHS) is the same expression with deviations from the warring equilibrium. The additive technology assumed

[9] See Harsanyi and Selten (1988). The (somewhat ad-hoc) original intuition behind risk dominance in 2x2 games is as follows. Assume each player does not know what the other player thinks she will do and they have uniform second order beliefs. Each player i, j can take one of two actions, α, β and there are two symmetric equilibria. Call $\bar{s}_j$ the probability agent i playing action α_i that makes agent j indifferent between her actions. Higher s_j makes playing α_j optimal. Hence, the higher $\bar{s}_j$, the larger the range of beliefs that rationalize playing β_j. Equilibrium β risk-dominates equilibrium α if $\bar{s}_j + \bar{s}_i \geq 1$, that is, if the sum of ranges of second order beliefs across players that rationalize playing the β strategies is larger that the sum of the ranges for the α equilibrium.

[10] See Carlsson and Van Damme (1993)

in the payoff matrix allows for a simple expression for the threshold

$$\theta^{RD} = \frac{(S-W)^2 - (\Pi_A - M)(\Pi_B - M)}{(\Pi_A + \Pi_B) + 2(S - W - M)} \tag{3}$$
$$= \frac{(S-W-M)^2 - \Pi_A \Pi_B}{(\Pi_A + \Pi_B) + 2(S - W - M)} + M$$

Note that the LHS of equation (2) is increasing in θ while the RHS is decreasing. Hence, whenever $\theta > \theta^{RD}$, the LHS will be larger than the RHS, the peaceful equilibrium risk-dominates and groups can coordinate into peace. Whenever $\theta < \theta^{RD}$ they go to war. Hence, peace fails when there are bad economic shocks. This prediction is well supported by the data. See, for instance, Miguel et al (2004) for the case of civil wars in Africa.

The first important characteristic of this equilibrium is that strategic risk is the direct cause of conflict for a range of realizations of the state of the economy. Since θ^{RD} is always above $\underline{\theta}$, for $\theta \in \left[\underline{\theta}, \theta^{RD}\right]$ groups fight even though peace would be sustainable as an equilibrium outcome with common knowledge. For any signal that a player receives, she knows that her opponent has received a signal that is very close, but it can be either above or below. The player balances the risk of attack against the opportunity cost. At low levels of θ, the small returns to play peace are not worth the risk of being caught off guard. Worse, each player knows that the other player is thinking in similar terms and hence both of them end up attacking. The threshold sits at the point where the expected returns of the economy are high enough that peace is worth the risk. As σ converges to 0, miscoordination happens with vanishing probability but it still pins down the unique threshold because strategic risk is present all the way to the limit. Hence this game features the strategic risk that pervades relationships in weak states and the process of second-guessing the opponent is central in determining the equilibrium.

To gain more precise intuition on risk-dominance, note that equation (2) can be rewritten as

$$\prod_{i \in \{1,2\}} \frac{-W - \theta + S}{\Pi_i + \theta - M} = 1 \tag{4}$$

To make sense of this expression, consider the point of view of player i, and assume that she puts a subjective probability p on the fact that player $-i$ chooses to be peaceful. Then player i chooses to be peaceful herself whenever,

$$p(\Pi_i + \theta) + (1-p)(\theta - S) \geq pM - (1-p)W \iff \frac{-W-\theta+S}{\Pi_i + \theta - M}\frac{1-p}{p} \leq 1$$

Keeping p constant, player i balances the potential loss $-W-\theta+S$ from being peaceful when her opponent is aggressive and the potential loss $\Pi_i + \theta - M$ from being aggressive when her opponent is peaceful. Thus the ratio $\frac{-W-\theta+S}{\Pi_i+\theta-M}$ summarizes the relative losses of playing peace when the opponent deviates. Intuitively, in the presence of uncertainty, the greater this ratio is, the more aggressive player $-i$ will expect player i to be. In other words, keeping constant the second beliefs of player $-i$ -what $-i$ thinks that i thinks that $-i$ will do, captured by p- she will expect higher propensity of i to attack when this ratio is high.

When player $-i$ decides whether to attack or not, she will balance this measure of the aggressiveness of i with her own relative gains of attacking, $\frac{-W-\theta+S}{\Pi_{-i}+\theta-M}$. Hence the strategic risk that both players face can be measured by an increasing function of both $\frac{-W-\theta+S}{\Pi_i+\theta-M}$ and $\frac{-W-\theta+S}{\Pi_{-i}+\theta-M}$. For player i the first ratio is the relative loss of paying peace and being betrayed while the second ratio measures the relative probability of a betrayal. The converse is true for player $-i$. We can now give an intuitive interpretation of the product.

$$\underbrace{\overbrace{\frac{-W-\theta+S}{\Pi_i+\theta-M}}^{\text{relative likelihood of a deviation by } -i}}_{\text{relative cost of mistake to } i} \times \underbrace{\overbrace{\frac{-W-\theta+S}{\Pi_{-i}+\theta-M}}^{\text{relative cost of mistake to -}i}}_{\text{relative likelihood of a deviation by } i}$$

Since this expression is symmetric, when this product is high both players consider that the risks inherent in playing peace are too high and the war equilibrium is risk-dominant. As a consequence, players might engage in unnecessary conflict, either because they face large losses should the other player be aggressive, or because the other player is actually likely to be aggressive. This expression is thus a measure of the aggregate strategic risk involved in playing peace.[11]

[11] Harsanyi and Selten (1988) show that when both players have uniform second order beliefs, the unique equilibrium is exactly selected by this expression. This is a somewhat ad-hoc justification as there is no compelling reason for uniformity in second order beliefs. However, as noted above, Carlsson and Van Damme (1993) microfounded this selection criterion with noisy observations of a common state of the world. This natural departure from common knowledge selects risk-dominance in the context of a well-defined non-cooperative game.

It is useful to note at this stage that from differentiating equation (2) we obtain,

$$-1 < \frac{\partial \theta^{RD}}{\partial \Pi_i} < 0 \tag{5}$$

2.3 Comparative Statics

Once we have a simple model of conflict as a result of strategic risk, it is interesting to examine how the underlying economic and technologic variables affect the propensity to violence. We can do this by examining the comparative statics of θ^{RD}. Straighforward differentiation of expression (3) establishes the following proposition.

Proposition 1. *The risk-dominant threshold θ^{RD} is increasing in $S - W$, increasing in M and decreasing in Π_i.*

Thus, the higher the temptation to attack M and the greater the fear of being attacked, S the more likely is war to happen. A high W implies that payoffs upon war are punitive. This clearly helps groups to coordinate into peace. These are variables related to the technology of war. High S, for instance, can be associated with a high propensity to victimize the civilian population.

Finally, the higher are the returns to peace for any group, the better are the prospects for peace. These comparative statics are intuitive as players fear being caught unprepared. A high Π_i means that it is common knowledge that player i has a lot to lose by attacking. This allays the fears of player j and helps both of them to sustain peace.

These comparative statics are natural and intuitive. Note that other typically used equilibria do not display them. In particular, the Pareto-optimal equilibrium under common knowledge is defined by coordinating into peace as much as possible, that is, by using $\underline{\theta}$ as threshold. It is immediate to check that $\underline{\theta}$ does not depend at all on the fear of being attacked, indexed by $S - W$. This is natural because under common knowledge there is no room for miscoordination. The shadow of a coordination failure in the risk-dominant equilibrium formally captures the intuition of Hobbes and the Realist school of international relations and enriches it by showing why failure is not ever-present: when the opportunity cost of war is high, groups can escape the trap of fear.

Inequality. The risk-dominace threshold features an intriguing characteristic: it is sensitive to inequality. To see this, normalize the sum $\Pi_A + \Pi_B = \Pi$ and define λ such that

$\Pi_A = \lambda\Pi$ and $\Pi_B = (1-\lambda)\Pi$. The threshold θ^{RD} is expressed as,

$$\theta^{RD} = \frac{(S-W-M)^2 - \lambda(1-\lambda)\Pi}{\Pi + 2(S-W-M)} + M \tag{6}$$

This expression is minimized at $\lambda = 1/2$, which implies that increasing inequality increases the probability of conflict. This is a specific property of the risk-dominant equilibrium and the mathematical motive lies in the multiplicative nature of the risk-dominance calculation, as shown in expression (4). By increasing inequality in wealth, we make two things common knowledge. First that group A is now richer and reluctant to go to war. Second, that group B is now poorer and thus becomes less conservative. Because of the risk-dominance calculation, the newfound reluctance by group A cannot compensate fully for the aggressiveness of group B, and as a consequence the aggregate risk of violence increases.

Note that we are keeping total income constant in this comparative statics. This is important because we already know from proposition 1 that θ^{RD} is decreasing in Π_i for $i \in \{A, B\}$. Hence, increasing income for any of the groups can only be conducive to increased peace. Note that this implies that increasing the income of the rich reduces violence. This is the case because there is *common knowledge* that the rich have become more reluctant to use violence. Since in this case there is no countervailing force from the poor becoming poorer, it allows both groups to coordinate into a larger range of peace. It follows that not any increase in inequality is conducive to violence in this model. For this to be the case, it is a necessary condition that the poor group becomes poorer.

However, this does not imply that any increase in aggregate income increases the chances to coordinate into peace. An increase in total income that comes at a cost for some groups can increase the probability of violence as expression (6) shows. Hence, when redistribution mechanisms are not in place, some policies that provide lopsided gains may increase social instability and violence.[12]

The fact that θ^{RD} is decreasing in both players' wealth does not support the intuition according to which the richer is my neighbor, the more I want to loot him. By keeping M constant, we isolate the effect of inequality on the capacity of groups to coordinate. In fact differences in income may change M, S and W. And depending on the circumstances, these variables may be increasing or decreasing in wealth. For instance, a richer group may

[12]For instance, if we believe in a basic Hecksher-Olin model of trade, opening to trade may be beneficial in developing countries to the extent that it both increases wealth and reduces inequality. However this result also highlights the fact that the stabilization benefits of trade openness will only be realized if its proceeds are relatively equally distributed.

promise higher proceeds from looting but at the same time it might be better armed. Hence M could be both increasing or decreasing on the wealth of the opponent.

In fact, the same analysis that we have done for income can be done for S, W and M. In particular, it is easy to see from (4) that starting from symmetry, inequality in M is conducive to violence but inequality in W and S increase stability. This is because W and S appear in the deviations from the war equilibrium and Π and M parametrize deviations from the peace equilibrium. Hence, if M is increasing in the income of the opponent -if looting is proportional to the wealth of the target- income inequality is doubly conducing to violence. Moreover, with this technology one can construct cases in which increasing the income of the rich would increase the prevalence of war because the effect present in proposition 1 is canceled by the increase in inequality in M.

The Effect of Deterrence. Proposition 1 shows that increasing W reduces θ^{RD} and thus the probability of violence. Hence, this model exhibits deterrence in the sense that increasing the costs of war always diminishes the probability of violence. However, increasing W also means that wars are more costly in terms of utility. Hence, the net expected utility effect of and increase in W is ambiguous. Intuitively, we would expect rich countries to benefit more from an increase in W and the associated reduction in the probability of war for two reasons: first, their underlying propensity of conflict is lower, and hence the increased costs of war are realized with lower probability.[13] Second, the proceeds of peace are higher in rich countries and they have greater value for peace. However, this simple intuition is obscured by the fact that the reduction in the probability of war caused by an increase in W will not be the same for countries with different Π.[14] To avoid these confouding effects, Lemmas 2 and 3 discuss deterrence while keeping the propensity for conflict constant.

Lemma 2. *For any real number r and any W, we consider the family of groups $\mathcal{F}_{r,W}$ characterized by $\Pi_A = \Pi_B = \Pi$ and $S + M = \Pi + r$. Then all groups in $\mathcal{F}_{r,W}$ have the same propensity to go to war : $\theta^{RD} = (r - W)/2$.*

This lemma allows us to consider changes in wealth that do not imply a change in the un-

[13]The intuition is simple: it is better not to give atomic devices to countries that face civil war with high probability because the marginal reduction in this probability will not compensate the huge costs of a realized nuclear war. On the contrary, for countries that almost never face violence, the costs of increasing their destructive capacity are seldom realized.

[14]It is immediate from (3) that $\frac{\partial \theta^{RD}}{\partial W}$ is a function of M, S and Π

derlying probability of war.[15] Keeping the likelihood of war constant, we can unambiguously compare the value of deterrence for rich and poor countries.

Lemma 3. *Consider a family $\mathcal{F}_{r,W}$. For any group in that family denote V the limit value of playing game Γ_σ as σ goes to 0. Then, we have*

$$\frac{\partial}{\partial \Pi}\left(\frac{\partial V}{\partial W}\right)_{S+M=\Pi+r} > 0 \tag{7}$$

That is, keeping the propensity to go to war constant, the value of deterrence increases with the wealth of the groups.

Moreover, there always exists Π large enough so that $\frac{\partial V}{\partial W} > 0$, and parameters W,r and Π small enough, $\frac{\partial V}{\partial W} < 0$.

This lemma shows that for identical levels of conflict, richer countries benefit from deterrence more than poor ones because of the higher benefits they enjoy from peace. If we add to this that rich countries have a lower underlying probability of conflict, it is clear that deterrence will always be welfare improving for countries that are rich enough. On the contrary, arms proliferation will have adverse consequences in underdeveloped countries. The implication is that while it is probably welfare improving to send aid to poor countries that are experiencing conflict, we should be wary of sending aid to relatively well off countries that choose to give way to violence.

3 War as an exit game

The one-shot game may underestimate the capacity of groups to avoid violence. If contenders are forward looking, they know that there are long-term costs to violence. For instance, in Macedonia, Slavs and Albanians could be potentially locked in a prisoners' dilemma situation in terms of current payoffs and first strike advantages. However, the possibility of joining the European Union in the future provides a strong incentive to coordinate into peaceful coexistence. In general, the actual value of peace is higher than the current payoffs groups realize because initiating violence can trigger a regime change into one of future widespread violence. The weight of the future is already emphasized in Fearon and Laitin (1996) to explain why ethnic groups cooperate more often than an anarchic world view would make

[15] The conditions for this lemma hold when, for instance, the amount plundered M varies one to one with wealth Π and destruction S is independent of wealth.

us expect. How can the one-shot coordination game be modified in order to capture the importance of the future?

We consider the dynamic exit game structure proposed by Chassang (2005). This framework allows for the extension of the Global Games information structure to a dynamic setting and permits the introduction of time varying capital stocks. Since the exit game only differs from a repeated game in that payoffs upon deviation are exogenously given, any strategic consideration of an equilibrium of the fully repeated game before the punishment phase can be acommodated.

In the context of conflict, the assumption that whenever a group attacks the game ends, does not seem inappropriate given that the median duration of a civil war is 6 years (see Fearon and Laitin (2003)). Indeed, the assumption of exit is in fact equivalent to the assumption that incentives that would occur conditionally on a war having taken place are unimportant. Given the amount of turmoil caused by civil conflict, this does not seem a restrictive assumption.

3.1 Global games information structures in exit games

The game we consider has two players $i \in \{A, B\}$ with action space $\{P, W\}$ and infinite horizon $t \in \{1, \ldots, \infty\}$. At time t, players get flow payoffs,

	P	W
P	$\Pi(k_{i,t}) + \theta_t$	$\theta_t - S$
W	M	$-W$

Where $k_{i,t}$ denotes the productive capital of group i at time t. Π is weakly concave and increasing in k. Capital follows an exogenously given, deterministic, recurrence equation. Denoting $\mathbf{k} = (k_i, k_{-i})$, we have,

$$\mathbf{k}_{t+1} = L(\mathbf{k}_t) \tag{8}$$

Where L is a continuous and increasing mapping. Hence, in the absence of conflict, this model exhibits (exogenous) economic growth.[16] We assume that L is such that any long run capital stock must belong to a compact range R.

Whenever a group chooses to go to war, the game stops and players get a continuation

[16] The structure can easily be adapted to stochastic evolution of capital stocks. For simplicity, we concentrate on the case of deterministic growth.

payoff equal to zero. The sequence of states of the world $\{\theta_t\}$ is i.i.d. with distribution f_θ and support in $\mathbb{R}$. Players get a signal on the state of the world $s_{i,t} = \theta_t + \sigma\varepsilon_{i,t}$. We denote by Γ_σ this dynamic game.

Definition 1 (histories)**.** *Because of the exit game structure, at any decision node, the histories of players take the form $h_{i,t} = \{s_{i,1}, k_{i,1}, \ldots, s_{i,t}, k_{i,t}\}$. We denote $\mathcal{H}$ the set of possible histories.*

A strategy x is a mapping from $\mathcal{H}$ to $\{P, W\}$. At any history $h_{i,t}$, we denote $V_i(h_{i,t})$ the expected value of playing the game for player i. We must have $V_i > -W$ since any player can guarantee this payoff by going to war.

As in the static game, we want to ensure that the best response to an attack is to attack. For this reason, we assume that $S > (1+\beta)W + M$. This is a sufficient condition to insure that no group accepts passively to be attacked.

A few definitions are required to present the selection results.

Definition 2. *We denote $\overline{\Pi} = \max_{k \in R} \Pi(k)$ and $\overline{V}$ an upper bound to the value of playing, for instance the value of playing the best equilibirum under common-knowledge.*

Definition 3 (order on strategies)**.** *We define an order, $\prec$, on strategies by,*

$$x \prec x' \iff \{\forall h \in \mathcal{H}, x(h) = P \Rightarrow x'(h) = P\}$$

Definition 4 (threshold form)**.** *A strategy x has a threshold form if and only if:*

1. *There exists a mapping $\tilde{x}$ such that for all $h_{i,t} \in \mathcal{H}$, $x(h_{i,t}) = \tilde{x}(s_{i,t}, \mathbf{k}_t)$*
2. *For all $\mathbf{k}_t$, there exists $\theta^T(\mathbf{k}_t) \in \mathbb{R}$ such that, $x(h_{i,t}) = P\mathbf{1}_{s_{i,t} > \theta^T(\mathbf{k}_t)} + W\mathbf{1}_{s_{i,t} \leq \theta^T(\mathbf{k}_t)}$*

Hence, a strategy takes a threshold form when it only depends on history through the current signal on the state of the economy and the current level of capital for both groups, and given the level of capital, there is a threshold in the state of the world above which groups play peace and below groups play war. With these definitions, we can solve the dynamic game.

Lemma 4. *The game Γ_σ satisfies the assumptions of Theorem 7 of Chassang (2005). This implies that for σ small enough, the set of rationalizable strategies of Γ_σ is bounded by a greatest and a smallest Nash equilibria with respect to order $\prec$. These equilibria, denoted*

$(x_A^{H,\sigma}, x_B^{H,\sigma})$ and $(x_A^{L,\sigma}, x_B^{L,\sigma})$ take a threshold form. These equilibria are associated with value functions upon continuation $\overrightarrow{V}^{H,\sigma}(\cdot) = (V^{H,\sigma}(\cdot)_A, V_B^{H,\sigma}(\cdot))$ and $\overrightarrow{V}^{L,\sigma}(\cdot) = (V_A^{L,\sigma}(\cdot), V_B^{L,\sigma}(\cdot))$. As σ goes to zero, $\overrightarrow{V^{H,\sigma}}$ and $\overrightarrow{V^{L,\sigma}}$ converge to the highest and lowest fixed points of an increasing and continuous mapping Φ defined by,

$$\text{(9)} \qquad \Phi(V_i, V_{-i})(\mathbf{k}_{t-1}) = \begin{pmatrix} \mathbf{E}\left[-W\mathbf{1}_{\theta<\theta_t^{RD}} + (\Pi_i(\mathbf{k}_t) + \theta + \beta V_i(\mathbf{k}_t)\mathbf{1}_{\theta>\theta_t^{RD}}\right] \\ \mathbf{E}\left[-W\mathbf{1}_{\theta<\theta_t^{RD}} + (\Pi{-}i(\mathbf{k}_t) + \theta + \beta V_{-i}(\mathbf{k}_t))\mathbf{1}_{w>\theta_t^{RD}}\right] \end{pmatrix}$$

Where θ_t^{RD} is defined by

$$\text{(10)} \qquad \theta_t^{RD} = \frac{(S-W-M)^2 - (\Pi_i(\mathbf{k}_t) + \beta V_i(\mathbf{k}_t))(\Pi_{-i}(\mathbf{k}_t) + \beta V_{-i}(\mathbf{k}_t))}{(\Pi_i(\mathbf{k}_t) + \beta V_i(\mathbf{k}_t)) + (\Pi_j(\mathbf{k}_t) + \beta V_j(\mathbf{k}_t)) + 2(S-W-M)} + M$$

An immediate corollary of this result is that equilibrium is unique at the limit whenever Φ has a unique fixed point. We now provide a sufficient condition under which this will be the case.

Lemma 5 (sufficient condition for uniqueness)**.** *Whenever the distribution of states of the world is such that,*

$$\text{(11)} \qquad \sup|f_\theta| < \frac{1-\beta}{4\beta(\overline{\Pi} + \beta\overline{V} + S)}$$

Then, the mapping Φ is contracting with a rate $\lambda < 1$. This implies that as σ goes to 0, the set of rationalizable strategies of game Γ_σ converges to a unique equilibrium associated with continuation value functions, (V_i, V_{-i}).

Assumption 1. *For the rest of the paper we consider a range of parameter values such that Φ is a contraction mapping*[17]

A close observation of expression (10) shows that the structure of the unique equilibrium is similar to the equilibrium in the static version of the game. In particular, the equilibrium behaves as if we applied the static Global Games structure each period to the following payoff matrix, which incorporates the fact that upon a peaceful interaction, the game continues into the future.

[17] If we relax this assumption, the game Γ_σ will have extreme equilibria with respect to order $\prec$ and the comparative statics presented in the remaining sections of the paper will hold for these extreme equilibria. Moreover, Chassang (2006) shows that the extreme equilibria of Γ_σ are stable in the sense of local dominance solvability, which - to an extent - justifies taking comparative statics, even though there are multiple equilibria.

$$
(12) \qquad \begin{array}{c|ll}
 & P & W \\
\hline
P & \Pi(k_{i,t}) + \theta_t + \beta V_i(\mathbf{k}_t) & \theta_t - S \\
W & M & -W
\end{array}
$$

This equilibrium is thus characterized by a sequence of thresholds $\{\theta_t^{RD}\}_{t\in\mathbb{N}}$. The threshold θ^{RD} depends on the contemporary stocks of capital and, through $V_i(\mathbf{k}_t)$ and $V_{-i}(\mathbf{k}_t)$, on the future path of the economy and the future probability of conflict. As before, at any given point in time, groups go to war if and only if the realized state of the world is below the threshold for that period. Hence, this model still exhibits a correlation between poor states of the world and the ocurrence of conflict.

3.2 Comparative statics with stationary capital

Many interesting comparative statics can be obtained in the case where capital is constant:

$$L(\mathbf{k}) = \mathbf{k}$$

The effect of wealth and military technology.

Proposition 2. *V_i is increasing in Π_i, Π_{-i} and decreasing in S and M. θ^{RD} is decreasing in Π_i and increasing in S and M.*

Most of the comparative statics of the one-shot game are maintained in the stationary game. Note, however, that the effect of a change in any of the parameters on the current probability of conflict is amplified because future probabilities of conflict and payoffs enter the expression for θ^{RD} via V_A and V_B.

Precisely because of this feedback, the comparative static with respect to W does not survive the introduction of dynamics. In the static model, the value function was not necessarily monotonous in W, however the threshold for peace was strictly decreasing in W. Now, since V_i affects the current probability of conflict, the effect of military technology on the probability of war also becomes ambiguous.

Next, we examine how inequality affects the likelihood of violence in the exit game.

Inequality and conflict.

Lemma 6. *Assume without loss of generality that $k_i < k_{-i}$, then*

$$\frac{\partial \theta^{RD}}{\partial k_i} < \frac{\partial \theta^{RD}}{\partial k_{-i}} \tag{13}$$

Moreover we have

$$\frac{\partial V_i}{\partial k_i} - \frac{\partial V_i}{\partial k_{-i}} > \frac{\partial V_{-i}}{\partial k_{-i}} - \frac{\partial V_{-i}}{\partial k_i} \quad \textit{and} \quad \frac{\partial V_i}{\partial k_i} - \frac{\partial V_i}{\partial k_{-i}} > 0 \tag{14}$$

This lemma shows that θ^{RD} is sensitive to inequality in the sense that an increase in one unit of capital is more beneficial if it increases the income of the poor rather than the rich. Moreover, the second part of the lemma shows that this is also true if we take a utilitarian view of welfare, as we can rewrite

$$\frac{\partial V_i}{\partial k_i} + \frac{\partial V_{-i}}{\partial k_i} \geq \frac{\partial V_{-i}}{\partial k_{-i}} + \frac{\partial V_i}{\partial k_{-i}}$$

which implies that it is always positive to reduce inequality. It is important to emphasize that this result does not depend on the concavity of Π. Thus, even when production is linear in capital, it is strictly better to target aid to the poor. This generalizes to dynamic games the fact that the risk-dominant threshold is increasing in inequality. In fact the impact of inequality on conflict is compounded in the dynamic exit game by entering V_i and V_{-i}.

Patience and Conflict. Now we turn to examine the effect of patience on the probability of resorting to violence. Because of the exit structure of the game, the way we model payoffs upon exit, M, W, S becomes important. We consider the following functional form:

$$W = \frac{1}{1-\beta} w \ ; \quad M = m - \frac{\beta}{1-\beta} w \ ; \quad S = s + \frac{\beta}{1-\beta} w$$

This form is akin to the payoffs that would accrue if the groups were playing a fully repeated game with grim-trigger strategies. The following proposition states the effect of patience on conflict.

Proposition 3. *With this specification of S, W and M, θ^{RD} is decreasing in β.*

The interpretation of this result is clear once we recall the structure of (12): forward looking groups take into account that there is an option value to play peace which is greater the more patient they are. This helps groups coordinate into peaceful coexistence as it is

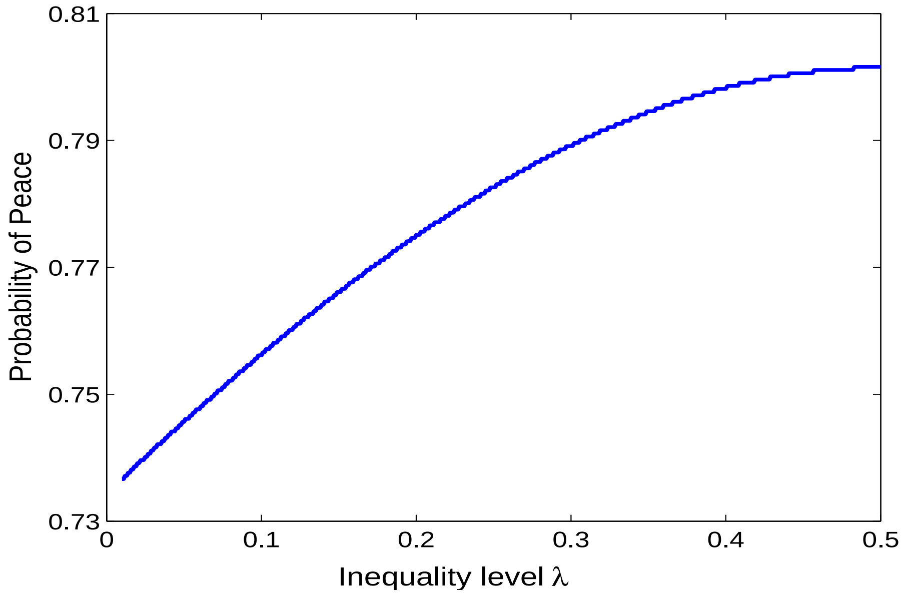

Figure 1: Probability of Peace as a function of inequality parameter λ. Uniform distribution over $[-8, 12]$, $k = 5$, $m = 0$, $w = 1$, $s = 5$, $\Pi(k) = k$.

common knowledge that the opportunity cost of violence is higher for everybody.

The next result establishes that this effect of patience is also increasing in wealth. In other words, patience and wealth reinforce each other in helping groups to coordinate into peace.

Lemma 7 (complementarity of patience and wealth)**.** *Consider symmetric groups with wealth k and discount rate β. Whenever $\frac{\partial f_\theta}{\partial \theta}(\theta^{RD}) \leq 0$ then*

$$\frac{\partial^2 \theta^{RD}}{\partial k_i \partial \beta} < 0 \tag{15}$$

and

$$\frac{\partial^2 V_i}{\partial k_i \partial \beta} > 0 \quad \textit{and} \quad \frac{\partial^2 V_i}{\partial k_{-i} \partial \beta} > 0 \tag{16}$$

Note that Lemma 7 implies that as long as $f'_\theta(\theta^{RD})$ is not too high, then $\frac{\partial^2 P(\theta > \theta^{RD})}{\partial k_i \partial \beta} > 0$. The intuition for this result is that forward looking agents experiment a time-multiplier effect. An increase in a unit of capital decreases the probability of conflict in all future periods. The value of these future changes is increasing in β, and thus the marginal effect

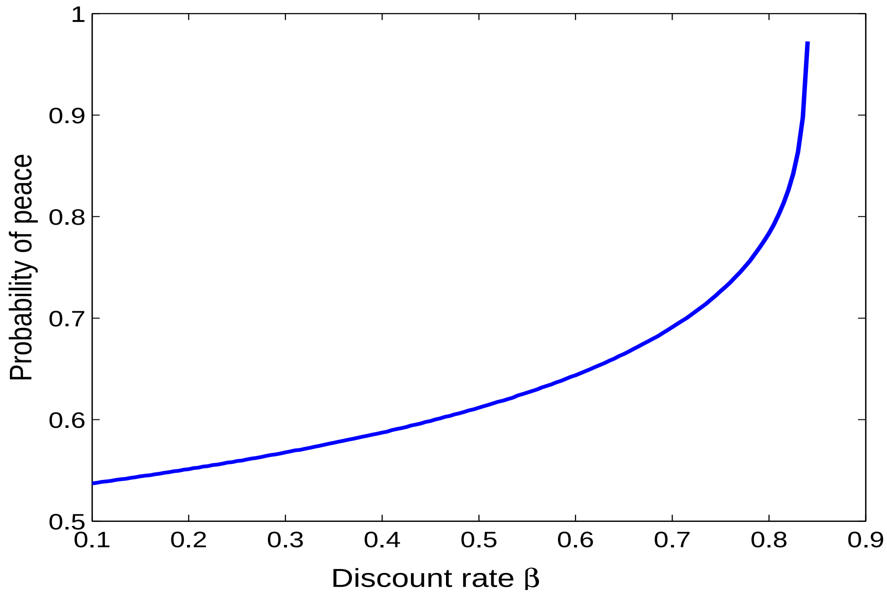

Figure 2: Probability of peace as a function of the discount rate β. Uniform distribution over $[-8, 12]$, $k = 1$, $m = 0$, $w = 1$, $s = 5$,$\Pi(k) = k$.

of capital on peace increases in the patience of citizens.[18]

In fact, in the presence of the possibility of conflict the players face an effective discount rate $\tilde{\beta} = \beta P(\theta^{RD} < \theta)$. Hence, any intervention that reduces θ^{RD} is complementary to patience as both increase the effective discount rate. Also, a corollary of Lemma 7 is that an increase in wealth is complementary to any intervention that increases the effective discount rate.

Next, we assume linear production technologies and consider scale effects of wealth on peace. Note that the linearity assumption can be viewed as a renormalization operation.

Lemma 8 (Gains of scale in wealth)**.** *Consider symmetric groups with wealth k and assume that $\pi(k) = k$. We consider jointly varying the wealth of these groups. Whenever $\frac{\partial f_\theta}{\partial \theta}(\theta^{RD}) \leq 0$ then*

$$\frac{\partial^2 \theta^{RD}}{\partial k^2} < 0 \tag{17}$$

[18]The reason why it suffices that $\frac{\partial f_\theta}{\partial \theta}(\theta^{RD}) \leq 0$ is that the amount by which the probability of war diminishes when the capital stock is increased also depends on where in the distribution of states of the world the current threshold for peace is. If by increasing β one shifts θ^{RD} to a zone where there is no mass, then Lemma 7 may not hold.

and

$$\frac{\partial^2 V_i}{\partial k^2} > 0 \tag{18}$$

The gains of scale in wealth result entirely from the dynamic structure: greater capital not only increases the value of continuation, it also increases the effective discount rate $\tilde{\beta}$ by making continuation itself more likely. Note that for $\beta = 0$ this effect disappears: $\frac{\partial^2 \theta^{RD}}{\partial k^2} = 0$. Hence at $\beta = 0$ we obtain that $\frac{\partial^3 \theta^{RD}}{\partial k^2 \partial \beta} < 0$. The more patient the players are, the greater the gains from scale in wealth will be.

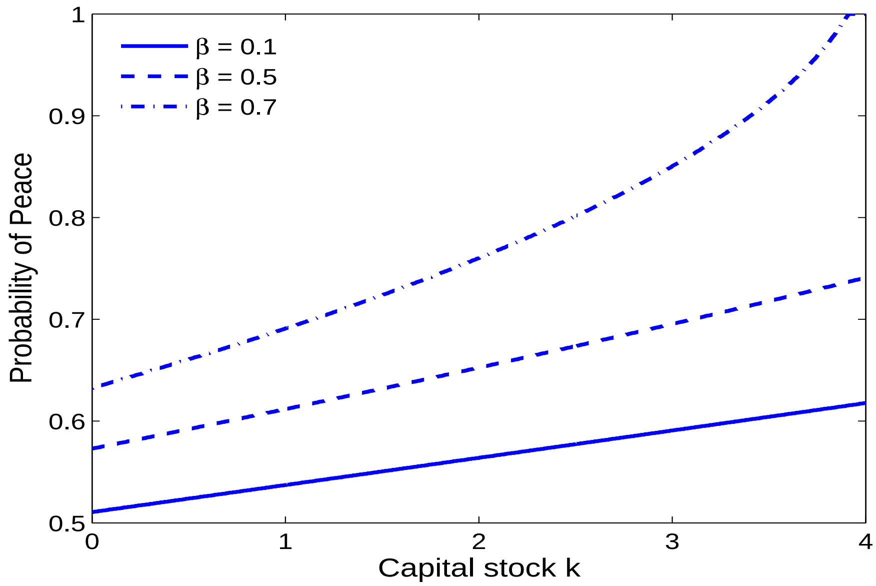

Figure 3: Probability of Peace as a function of capital stock k and discount rate β. Uniform distribution over $[-8, 12]$, $m = 0$, $w = 1$, $s = 5$, $\Pi(k) = k$.

Figures 3 and 4 show the effect of capital and patience on the current probability of peace and the value of the game. Following Lemma 7, the marginal impact of capital increases with β. One can read on Figure 3 how the likelihood of peace is increasing in k. The static impact of increasing the proceeds of peace can be gauged by comparing probabilities when β is close to 0. The returns to scale in wealth because of dynamic incentives are clear in the convexity of the curves for $\beta > 0$.

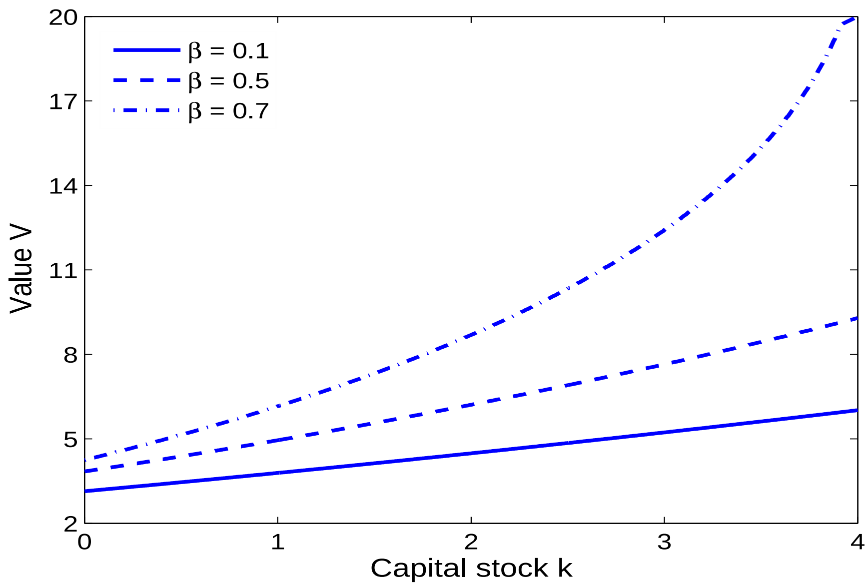

Figure 4: Value of playing as a function of capital stock k and discount rate β. Uniform distribution over $[-8, 12]$, $m = 0$, $w = 1$, $s = 5$, $\Pi(k) = k$.

3.3 Comparative statics with non-stationary capital stocks

Can the promise of entry in the European Union stem inter-communal violence in places like Macedonia or Turkey? How do expectations of future growth affect conflict? What about inequality in expected growth patterns? The exit game framework allows us to ask a variety of questions involving the effect of future growth on the current likelihood of violence.

First, we address the effect of future exogenous economic growth on the current probability of conflict.

Proposition 4 (growth and conflict)**.** *Index the growth process by a variable $z \in \mathbb{R}$: $\mathbf{k}_{t+1} = L_z(\mathbf{k}_t)$, such that, L_z is increasing in z. Then for any initial capital stocks $\mathbf{k}$, we have that*

$$\frac{\partial \theta_z^{RD}(\mathbf{k})}{\partial z} \leq 0 \quad \textit{and} \quad \frac{\partial V_z(\mathbf{k})}{\partial z} \geq 0 \tag{19}$$

Lemma 4 makes clear that increasing the slope of the growth process reduces the current propensity of violence. Hence, taking into account expectations of growth may explain variations in conflict propensity that do not correspond to observable variations in the current state of the economy. This may help explain the largely peaceful assimilation of spanish immigrants into Catalonia and the Basque Country in the 1960s, even though a "sons of the soil" dynamic could have started: this massive immigration preceded a period of robust economic growth.

Lemma 9 (complementarity of patience and growth)**.** *Consider symmetric groups, with a commmon capital stock following the recurrence equation* $k_{t+1} = L_z(k_t)$*. Denote* θ_t^{RD} *the threshold of peace at time* t*. Then whenever for all* $t \in \mathbb{N}$, $f'_\theta(\theta_t^{RD}) \leq 0$*, we have,*

$$\forall t \in \mathbb{N}, \quad \frac{\partial^2 \theta_t^{RD}}{\partial z \partial \beta} < 0 \tag{20}$$

Lemma 9 simply indicates that the time-multiplier effect is also at work when we examine the role of growth. This is not surprising, as the mechanism is basically the same as in the stationary model: the increased future economic returns and reduced probabilities of conflict compound into a reduced current propensity of violence via their effect on V.

Now we turn to the impact of unequal sharing of the proceeds of growth within a country.

Lemma 10 (inequality in growth)**.** *Assume that each group's capital stock follows it's own growth process, that is,*

$$\forall\, i \;\in \{1, 2\}, \quad k^i_{t+1} = L_{z_i}(k^i_t) \tag{21}$$

With L *increasing and weakly concave in both* k *and* z*.*
Then, whenever $k_i \leq k_{-i}$ *and* $z_i \leq z_{-i}$*, we have,*

$$\frac{\partial \theta^{RD}(\mathbf{k})}{\partial z_i} \leq \frac{\partial \theta^{RD}(\mathbf{k})}{\partial z_{-i}} \tag{22}$$

Moreover we have

$$\frac{\partial V_i}{\partial z_i} - \frac{\partial V_i}{\partial z_{-i}} > \frac{\partial V_{-i}}{\partial z_{-i}} - \frac{\partial V_{-i}}{\partial z_i} \quad \text{and} \quad \frac{\partial V_i}{\partial z_i} - \frac{\partial V_i}{\partial z_{-i}} > 0 \tag{23}$$

Lemma 10 shows that disparity in growth rates increases the propensity of conflict. For the same aggregate growth rate, a country that has all its groups enjoying the average rate of growth will be more peaceful than a country in which a group monopolizes economic growth.

This section underlines the fact that, keeping the current level of income constant, any expected future shock in income affects the current propensity of violence. This being the case, and assuming that joining the European Union provides widespread growth, we can conjecture that the promise of future adhesion may be a force at place in stemming communal violence in Eastern Europe and in the Balkans. On the contrary, an expected drop in the economic situation could fuel conflict well before the actual shock takes place. To put it starkly, to expect future conflict breeds current conflict.

4 The War Trap

The time preferences of citizens are clearly affected by the possibility of future conflict. When citizens judge the likelihood of violence to be high, they will put little value in future income whose realization is conditional on peace. This impact of violence on effective discount rates has adverse implications for the accumulation of savings in countries plagued by conflict. Our opportunity cost approach to conflict adds a feedback mechanism by which countries that don't manage to save also face a greater likelihood of conflict.

We first address the question of effective time preference in a setup where the two groups have constant capital stocks. The following lemma computes the substitution rate at which groups value the addition of an extra unit of capital at different points in time.

Lemma 11. *Consider two groups with constant capital stocks : $\forall t, k_{i,t} = k_i$. Denote $V_{i,1}$ the value of group i at period 1, before the state of the world is revealed. The effective intertemporal substitution rate of capital is,*

$$\rho \equiv \frac{\frac{\partial V_{i,1}}{\partial k_{i,2}}}{\frac{\partial V_{i,1}}{\partial k_{i,1}}} = \beta P(\theta \geq \theta^{RD}) - \beta \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} f_\theta(\theta^{RD})[\Pi_i(k_i) + \beta V_i + \theta^{RD} + W] \tag{24}$$

Condition (11) ensures that $\rho < 1$. Note that the first summand in (24) is in fact the effective discount rate $\tilde{\beta}$. Since we know that $\frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} < 0$, it follows that $\rho > \tilde{\beta}$. However, note that it is in fact possible to even have $\rho > \beta$. The reason for this counterintuitive possibility is that two forces ar at work. On the one hand, the probability of peace is smaller than one, which reduces the expected returns from investment, and this is captured by the first summand as $\tilde{\beta} < \beta$. However, on the other hand, increasing tomorrow's payoff increases the likelihood of peace for today *and* tomorrow. How those two effects balance out is ambiguous. However, because rich countries are less likely to go to war, the natural intuition is that effective discount rates will be increasing in wealth. The following lemma gives conditions under which this intuition holds.

Lemma 12. *Consider groups with identical and constant capital stocks k and such that $f'_\theta(\theta^{RD}) \leq 0$. Then the intertemporal substitution rate of capital is increasing in wealth, that* *is*, $\frac{\partial \rho}{\partial k} > 0$.

Lemma 12 is of interest for two reasons. First, it shows how a war trap may exist even in the presence of a central planner. In impoverished countries, the likelihood of conflict is very high. This reduces optimal investment rates, which in turn guarantees that the country will not rise out of poverty which further fuels conflict. This paints a situation of economic stagnation as a prelude to violent conflict. In that case even if a good string of states of the world allows peace to survive for a few periods, it will fail to generate the economic growth that is needed for the hazard rate of violence to diminish in the long term.

An additional effect appears when investment decisions are decentralized. Lemma 12 implies that when conflict is a possibility, there will be positive externalities at the investment stage. As is well known, in the presence of such externalities investment will be inefficiently low, worsening the war trap. Furthermore, those inefficiencies are likely to be greater in poor countries than in rich countries : since rich countries are unlikely to experience violent conflict in the first place, the inefficiency that results from players not taking into account that their investment reduces the likelihood of conflict is very limited; for poor countries on the other hand, the large impact of investment on the probability of conflict makes the collective action problem might be much more critical.

We provide a simple examination of these ideas in section 4.1, although a full fledged study of endogenous investment is beyond the scope of this paper.

4.1 A Simple Example with Endogenous Investment

We consider a country with symmetric groups and enrich the former model by having capital follow a simple recurrence equation: $k_{t+1} = (1-\delta)k_t + d$, where d is a simple investment decision $d \in \{0, I\}$, associated with costs $C(0) = 0$ and $C(I) = C$. We also assume that the distribution of states of the world is uniform.

A unique investor. Let us first consider the case in which investment is made by a unique investor that takes into account her impact on conflict and has the possibility to commit to future investments. Assume that she obtains a benefit of the form Ak from holding a capital stock k. Her value function W satisfies the Bellman equation,

$$W(k_t) = \max_{d(\cdot)\in\{0,I\}^{\mathbb{R}}} Ak_t - C(d) + \beta Proba\left(\theta > \theta^{RD}_{d(\cdot)}(k_t)\right) W(k_{t+1}) \tag{25}$$

We underline the fact that the investor takes into account that $\theta^{RD}_{d(\cdot)}$ depends on her policy function $d(\cdot)$. From Lemma 12 we know that her value for additional investment increases with her capital stock. This implies that her optimal investment policy will take a threshold form. More precisely, there exists a threshold k^* such that her optimal investment rule is,

$$d(k) = \begin{cases} 0 & \text{if} \quad k < k^* \\ I & \text{if} \quad k \geq k^* \end{cases}$$

Whenever $k^* > 0$ there will be multiple steady states. One of them can be characterized as a war trap. More precisely, if $k_0 < k^*$, then conditionally on peace $\lim_{t \to +\infty} k_t = 0$. The country does not experience economic growth and hence the probability of conflict remains high in every period.

Note that in this setting, the hypothesis that the investor can commit to future investments is not binding. Whenever she finds it optimal to invest today, she will find it optimal to invest thereafter.

Multiple investors. We now assume that investment decisions are made by a continuum of investors. This implies that an investor will take the probability of war as given when making her investment decisions. Given a threshold function $\theta^{RD}(k)$, an investor's value function W satisfies the Bellman equation,

$$W(k_t) = \max_{d \in \{0,I\}} Ak_t - C(d) + \beta Proba(\theta > \theta^{RD}(k_t))W(k_{t+1}) \tag{26}$$

It is possible for this game to multiple equilibria. The exit game framework guarantees however that they have a relatively simple structure. An equilibrium of the current game is characterized by a policy function $d(\cdot)$ from investors and a conflict threshold function θ^{RD} from the two groups. Definition 2 introduced a partial order denoted $\prec$ on peace and war decisions. We now introduce a natural order on investment decisions:

$$d \lhd d' \iff \forall k, d(k) \leq d'(k)$$

It is straightforward to show that the game between the two groups groups and the investors exhibits monotonous best-responses with respect to $\prec$ and $\lhd$. Thus the set of rationalizable strategies is bound by two extreme equilibria (d_H, θ^{RD}_H) and (d_L, θ^{RD}_L), such that for all k,

$$d_H(k) \geq d_L(k) \quad \text{and} \quad \theta_H^{RD}(k) < \theta_L^{RD}(k)$$

Moreover, these extreme equilibria can be obtained by iterating the best response mapping starting from the highest and the lowest possible pairs of strategies. It follows from this iteration process that the extreme equilibria are such that:

1. There exist $k^H < k^L$ such that

$$d_H(k) = I \iff k \geq k^H \quad \text{and} \quad d_L(k) = I \iff k \geq k^L$$

2. $\theta_H^{RD}(\cdot)$ and $\theta_L^{RD}(\cdot)$ are both increasing in k.

Finally, note that even in the high equilibrium, investment levels are lower than the socially efficient investment level, that is $k^H > k^*$: decentralized investors do not take into account the positive externality they have on each other by making peace more likely.

Again, as long as $k^H > 0$, the model with multiple investors will exhibit war traps. Countries that happen to begin with capital levels below k^H experience no growth due to the high probability of conflict that poverty entails. But obviously, even if peace remains through a good string of states of the world, the probability of conflict is not reduced and the country cannot grow out of the trap to a better steady state in which economic growth generates a steady reduction of the threat of conflict. On the contrary, for countries that have a initial level of capital above k^H, the hazard rate is diminishing in the length of the period of peace, which reinforces investment and growth, thus accelerating the process.

5 Policy Recommendations for Intervention Strategies

From our analysis of conflict as coordination failure we can draw a number of policy relevant implications. There are two types of interventions that we observe in reality and that we are interested in discussing: the first is economic aid in its various forms, the second is peace keeping interventions in which soldiers from a third party are deployed between groups in potential conflict. With respect to economic aid, we make four points.

First we should be cautious about providing aid conditional on war ocurring as this may have the effect of reducing W which increases the likelihood of war. War relief has an unambiguously positive effect in really poor countries but may actually make relatively wealthy countries worse off.

Second, reducing inequality across groups within a country reduces the incidence of violence. This is a direct consequence of Lemma 6. Thus, within a conflict zone, donors should direct their transfers conditional on peace to the poorest group since it gives the greatest returns in terms of peace keeping.

Third, it is important to note that the previous point does not necessarily apply accross conflict zones. We envisage the donor community as having to decide the allocation of funds across a number of conflict areas with symmetric groups that are locked in potential conflict. In such a situation, it may not be the best use of limited funds to target transfers to the poorest conflict area in the sample. The reason is that the effect of increasing capital on the probability of peace are non-linear, especially if citizens are patient. This convexity is apparent, for instance, in figures 5 and 6. For the parameter values represented by these figures it is clear that an extra unit of capital given to a country that has $k = 2$ obtains better returns than given to a country with $k = 0$. Moreover, the existence of increasing returns over a range of capital implies that the optimal allocation does not entail spreading aid across countries even if they have the same level of income. Concentrating on a case at a time yields higher global reduction in the incidence of coordination failures.

Finally, if countries differ in the degree of effective patience that their citizens exhibit, aid should be directed to the most patient countries. This is a corollary from Lemma 7 that can be appreciated in Figures 3 and 4. The intuition behind this result was discussed above. Note that differences in effective patience can reflect differences in baseline discount rates, in the disease environments, in the baseline probability of war or even in the security of property rights.

The model also sheds light on the role of peace keeping interventions. Peace-keeping operations are sent to mediate between contenders that have already reached a cease fire or truce of some sort. The focus of our framework on the expected duration of peace makes the exit structure especially adequate for this analysis.

First, note that in a completely stationary world a temporary intervention seems pointless: the probability of conflict is constant and hence whatever the consideration that prompted the intervention, it should also keep it in place. In other words, either a permanent intervention is optimal or there should be no intervention at all. However, the exit game with economic growth provides a rationale for a temporal intervention: as we have seen, the probability of conflict conditional on the time length of peace is diminishing because of the accumulation of capital in times of peace. Hence, forcing the contenders not to fight for a small number of periods may have important permanent welfare returns. Eventually the probability of peace is close enough to 1 and the returns to each additional period of intervention decline, which provides a rationale for finite time interventions.

Second, since peace keeping interventions only make sense in a context where peace permits some amount of economic growth, peace keeping interventions should be accompanied by measures encouraging investment. Those are in fact complementary instruments. Peace intervention make investment subsidies more effective and investment subsidies increase the long term impact of peace keeping interventions.

Finally, endogenizing investment highlights that it is essential that a peace intervention be able to commit to stay for a minimum amount of time. Otherwise, such interventions may have no impact on investment. With endogenous investment and i.i.d. states of the world, a lucky string of good states of the world will have no effect on future peace whereas the same number of peaceful periods guaranteed *ex-ante* by a peace keeping intervention may trigger investment which can lift the country out of the war trap.

6 Conclusion

In this paper we have presented a theory of intercommunal conflict that takes seriously the coordination problem central to the Security Dilemma. To emphasize the fear of being caught off guard as a driving force in this story, we focus on the risk-dominant equilibrium, which is a natural equilibrium concept when strategic risk considerations weigh heavily in the decision making process.

We show that the risk-dominant equilibrium displays behavior consistent with the agents trying to balance out the opportunity cost of violence with the fear of being attacked. In particular, the likelihood of conflict increases when the country is poor, the proceeds from looting are high and the offensive advantage large. The equilibrium also exhibits deterrence in the sense that reducing the payoffs in the situation of open conflict helps sustain peace. In addition, we show that inequality in income across groups is also conducive to violence while inequality in the offensive advantage is good for coordination into peaceful coexistence.

Besides this static model, we analyze a dynamic extension in which the game continues until a group defects and resorts to violence. This model allows us to examine the weight that the future has into current coordination. The value of peace increases because it contains the option value of continuing the game. This value is increasing in future economic growth and in patience. Hence, any expected positive future shock has effects in the current ability of groups to coordinate into peace. In reverse, expecting a bad economic situation for the future may trigger conflict today.

This dynamic version of the model allows us to extend the analysis to endogenous investment, unveiling the existence of a war trap. This is a situation in which poor countries do not invest because they expect conflict with high probability, which reinforces violence precisely because of the absence of economic growth. On the contrary, middle income countries can grow out of conflict by investing.

Finally, from the analysis, we draw policy prescriptions along two dimensions. First, the model provides a framework to discuss in which countries would foreign economic aid be most helpful. These need not be the poorest countries, because the time-multiplier effect of the future induces increasing returns to capital. Second, the model provides a rationale for the use of temporary peace-keeping operations when times of peace are accompanied by sufficient economic growth. It also suggests that investment enhancing measures would be strategic complements to peace-keeping operations.

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7 Appendix

Proof of Lemma 1. See Carlson and van Damme (1993). ■

Proof of Proposition 1. Recall equation (2). At θ^{RD} both equilibria exist which means that each of the factors on both sides are positive. Hence the left hand side is increasing in θ and Π_i and decreasing in M. The right hand side is decreasing in θ, Π_i and k and increasing in $S - W$. The comparative statics stated in the lemma are a direct consequence of these facts. ■

Proof of Lemma 2. Replace in expression (2): $\theta^{RD} = (S + M - W - \Pi)/2 = (r - W)/2$. ■

Proof of Lemma 3. We have

$$V = -WF(\theta^{RD}) + \int_{\theta^{RD}}^{+\infty} (\Pi_i + \theta) f_\theta d\theta$$

Hence,

$$\frac{\partial V}{\partial W} = -\frac{\partial \theta^{RD}}{\partial W} f_\theta(\theta^{RD})[W + \Pi_i + \theta^{RD}] - F(\theta^{RD})$$

Since we know that $\theta^{RD} = (r - W)/2$, we obtain,

$$\frac{\partial V}{\partial W} = \frac{1}{4} f_\theta \left(\frac{r - W}{2} \right) [2\Pi + W + r] - F\left(\frac{r - W}{2} \right) \tag{27}$$

Thus,

$$\frac{\partial}{\partial \Pi} \left(\frac{\partial V}{\partial W} \right)_{S+M=\Pi+r} = \frac{1}{2} f_\theta \left(\frac{r - W}{2} \right)$$

Finally since $f_\theta(\frac{r-W}{2}) > 0$ and $F(\frac{r-W}{2}) > 0$, it's clear from expression (27) that for Π large enough, $\frac{\partial V}{\partial W} > 0$ and that for r, Π and W small enough, $\frac{\partial V}{\partial W} \sim -F(0) < 0$. ■

Proof of Lemma 4. See Chassang (2005). ■

Proof of Lemma 5. Consider a function $\mathbf{g} = (g_i, g_{-i})$, mapping $\mathbb{R}^2$ onto $\mathbb{R}^2$. We define the norm of $\mathbf{g}$ by $||\mathbf{g}|| = ||g_i||_\infty + ||g_{-i}||_\infty$. Let us show that condition (11) implies that Φ is a contraction mapping with rate $\lambda < 1$.Consider $\mathbf{V}$ and $\mathbf{V}'$ two possible pairs of continuation value functions and denote $\mathbf{W}$ and $\mathbf{W}'$ their respective images by Φ. From equation (9), we have,

$$||\mathbf{W}-\mathbf{W}'|| \leq 4||f_\theta||_\infty \left|\frac{\partial \theta^{RD}}{\partial V}\right| ||\mathbf{V}-\mathbf{V}'||(\overline{\Pi}+\theta^{RD}+\beta\overline{V}+W)+\beta P(\theta > \theta^{RD})||\mathbf{V}-\mathbf{V}'||$$

Equation (5) implies $\left|\frac{\partial \theta^{RD}}{\partial V}\right| < \beta$, since in addition $\theta^{RD} < S-W$, we get that, $||\mathbf{W}-\mathbf{W}'|| \leq \lambda||\mathbf{V}-\mathbf{V}'||$, with

$$\lambda = 4\beta||f_\theta||_\infty(\overline{\Pi}+S+\beta\overline{V})+\beta < 1$$

This implies that Φ is a contraction mapping and that equilibrium is unique. ■

Proof of Proposition 2. Given continuation values V_i, V_{-i}, the equation defining the risk dominant threshold is

$$(\Pi_A+\beta V_A+\theta^{RD}-M)(\Pi_B+\beta V_B+\theta^{RD}-M) = (S-W-\theta^{RD})^2 \tag{28}$$

At θ^{RD}, $\Pi_i+\beta V_i+\theta^{RD}-M > 0$ and $S-W-\theta^{RD} > 0$, thus the left hand side is increasing in θ, Π_i, βV_i and decreasing in M, while the right hand side is decreasing in θ, and increasing in $S-W$. Therefore, θ^{RD} is increasing in M and S and decreasing in Π_i, V_i and W. We thus get that the first part of Lemma 2 implies the second. We can consider V_i as a function of Π_i, Π_{-i}, M, W, S and the mapping Φ as some mapping from bounded value functions to bounded value functions. From Assumption 1, Φ is a contraction mapping over the range of parameters we are concerned with. To any vector of functions $\overrightarrow{V}$ increasing in Π_i, decreasing in S and M, Φ associates a vector of functions that's also increasing in Π_i, decreasing in S and M. By iteratively applying Φ, this gives us the first part of the lemma. ■

Proof of Lemma 6. V_i and V_{-i} can be seen as functions mapping (k_i, k_{-i}) to real numbers. Those functions can be computed by iterating the contraction mapping Φ starting from some initial vector of functions $\overrightarrow{V}^0$. This iterative process produces sequences of value functions $\{\overrightarrow{V}^k\}_{k\in\mathbb{N}}$ and thresholds $\{\theta^{RD,k}\}_{k\in\mathbb{N}}$. We prove Lemma 6 by showing that when the initial value functions $\overrightarrow{V}^0$ *weakly* satisfy inequality (14), then all elements of $\{\theta^{RD,k}\}_{k\in\mathbb{N}}$ *strictly* satisfy (13) and all elements of $\{\overrightarrow{V}^k\}_{k\in\mathbb{N}}$ *strictly* satisfy (14).
We first begin by showing that if $\overrightarrow{V}^k$ weakly satisfies (14) then $\theta^{RD,k}$ strictly satisfies (13).

Recall the equation defining θ^{RD}:

$$(\theta + \Pi(k_i) + \beta V_i - M)(\theta + \Pi(k_{-i}) + \beta V_{-i} - M) = (S - \theta - W)^2$$

Differentiate this equation with respect to k_i. We obtain that,

$$\begin{aligned}
[\Pi(k_i) + \beta V_i + \Pi(k_{-i}) + \beta V_{-i} + 2(S - M - W)]\frac{\partial \theta}{\partial k_i} = &-\frac{\partial(\Pi + \beta V_i)}{\partial k_i}(\theta + \Pi_{-i} + \beta V_{-i} - M) \\
&- \beta\frac{\partial V_{-i}}{\partial k_i}(\theta + \Pi_i + \beta V_i - M)
\end{aligned}$$

Recall that by assumption $[\Pi(k_i) + \beta V_i + \Pi(k_{-i}) + \beta V_{-i}]$ is positive. It's also symmetric in i and $-i$. Therefore to prove that $\theta^{RD,k}$ satisfies inequation (13), we only need to prove that,

$$\begin{aligned}
\Delta \equiv &-\frac{\partial(\Pi(k_i) + \beta V_i)}{\partial k_i}(\theta + \Pi(k_{-i}) + \beta V_{-i} - M) - \beta\frac{\partial V_{-i}}{\partial k_i}(\theta + \Pi(k_i) + \beta V_i - M) \\
&+ \frac{\partial(\Pi(k_{-i}) + \beta V_{-i})}{\partial k_{-i}}(\theta + \Pi(k_i) + \beta V_i - M) + \beta\frac{\partial V_i}{\partial k_{-i}}(\theta + \Pi(k_{-i}) + \beta V_{-i} - M) \quad < \quad 0
\end{aligned}$$

We can write,

$$\begin{aligned}
\Delta = &-[\theta + \Pi(k_{-i}) + \beta V_{-i} - M]\left(\Pi'(k_i) + \beta\frac{\partial V_i}{\partial k_i} - \beta\frac{\partial V_i}{\partial k_{-i}}\right) \\
&+ [\theta + \Pi(k_i) + \beta V_i - M]\left(\Pi'(k_{-i}) + \beta\frac{\partial V_{-i}}{\partial k_{-i}} - \beta\frac{\partial V_{-i}}{\partial k_i}\right)
\end{aligned}$$

From the fact that $[\theta + \Pi(k_{-i}) + \beta V_{-i} - M] > [\theta + \Pi(k_i) + \beta V_i - M] > 0$ and the fact that $\overrightarrow{V}^k$ satisfies (14), simple algebra shows that indeed $\Delta < 0$.

Let us now show that if $\theta^{RD,k}$ and $\overrightarrow{V}^k$ respectively satisfy inequalities (13) and (14) then $\overrightarrow{V}^{k+1}$ satisfies (14). By definition, $\overrightarrow{V}^{k+1} = \Phi(\overrightarrow{V}^k)$, thus,

$$\begin{aligned}
\frac{\partial V_i^{k+1}}{\partial k_i} - \frac{\partial V_i^{k+1}}{\partial k_{-i}} = &\left[\Pi'(k_i) + \beta\left(\frac{\partial V_i^k}{\partial k_i} - \frac{\partial V_i^k}{\partial k_{-i}}\right)\right] P(\theta^{RD,k} \leq \theta) \\
&+ f_\theta(\theta^{RD,k})(-W - \Pi_i - \theta^{RD,k} - \beta V_i^k)\left(\frac{\partial \theta^{RD,k}}{\partial k_i} - \frac{\partial \theta^{RD,k}}{\partial k_{-i}}\right)
\end{aligned}$$

By assumption, $\frac{\partial V_i^k}{\partial k_i} - \frac{\partial V_i^k}{\partial k_{-i}} \geq \frac{\partial V_{-i}^k}{\partial k_{-i}} - \frac{\partial V_{-i}^k}{\partial k_i}$ and $\frac{\partial \theta^{RD,k}}{\partial k_i} - \frac{\partial \theta^{RD,k}}{\partial k_{-i}} < 0$. This implies that indeed,

$$\frac{\partial V_i^{k+1}}{\partial k_i} - \frac{\partial V_i^{k+1}}{\partial k_{-i}} > \frac{\partial V_{-i}^{k+1}}{\partial k_{-i}} - \frac{\partial V_{-i}^{k+1}}{\partial k_i}$$

Applying Φ iteratively, the sequences $\{\mathbf{V}^k\}_{k\in\mathbb{N}}$ and $\{\theta^{RD,k}\}_{k\in\mathbb{N}}$ converge to the equilibrium $\mathbf{V}$ and θ^{RD}. Inequalities (13) and (14) hold weakly at the limit. In addition, the proof shows that if (13) and (14) hold weakly at a fix point, they must hold strictly by iteration of Φ. ■

To proof the next several results we need the following lemma:

Lemma 13. *The equilibrium behavior of the players is invariant with respect to shifts in the flow payoffs of the form:* $\tilde{w} = w - h$; $\tilde{s} = s - h$; $\tilde{m} = m + h$; $\tilde{\Pi}_i = \Pi_i + h$.

Proof of Lemma 13. Denote $\tilde{V}_i = V_i + \frac{1}{1-\beta}h$. Let us show that $\tilde{V}_i$ is a fixed point of $\tilde{\Phi}$. First note from expression (10) that with this $\tilde{V}_i$, we must have $\tilde{\theta}^{RD} = \theta^{RD}$. Finally, all payoffs in the expression of $\tilde{\Phi}$ are shifted by a term $\frac{1}{1-\beta}h$. Thus $\tilde{V}_i$ is the unique equilibrium continuation value of the new game with shifted payoffs. This implies that the equilibrium thresholds are indeed the same. ■

Proof of Proposition 3. Consider the case where $\beta = 0$. Using the notations of lemma 13, pick $h = w$, then $V_i + \frac{1}{1-\beta}h > 0$. Therefore, using Lemma 13, we can equivalently study players behavior in a game where $V_A(\beta = 0) > 0$ and $V_B(\beta = 0) > 0$ and $w = 0$.

As before, we use a proof by induction. Assume that $\overrightarrow{V}^k$ is positive and increasing in β. Note that $S - W$ is constant in β and that M is decreasing in β. Therefore, we know from expression (10) that the risk-dominant threshold is decreasing in β. From the expression of Φ, this implies that $\overrightarrow{V}^{k+1}$ is increasing in β. Now consider the particular sequence of continuation values started from $\overrightarrow{V}^0 = (V_A(\beta = 0), V_B(\beta = 0))$. Then for all k, $\overrightarrow{V}^k(\beta = 0) = (V_A(\beta = 0), V_B(\beta = 0))$. For this particular sequence, being increasing in β also implies that all values $\overrightarrow{V}^k$ are strictly positive which finishes the induction step. For all k, $\overrightarrow{V}^k$ is strictly positive and increasing in β and $\theta^{RD,k}$ is decreasing in β.

These properties hold at the limit. ■

Proof of Lemma 7. From Lemma 13, without loss of generality, we consider a game for which $W = 0$. As usual, we prove the result by iteratively applying the contraction mapping Φ. In particular we show that if $\overrightarrow{V}^0$ is symmetric and $\theta^{RD,0}$ and $\overrightarrow{V}^0$ satisfy conditions (15) and (16) then, $\theta^{RD,1}$ and $\overrightarrow{V}^1$ also satisfy conditions (15) and (16). Let us show that if $\overrightarrow{V}^1$

satisfies condition (16) then $\theta^{RD,1}$ satisfies inequality (15). Indeed, we have that,

$$[\Pi(k_i) + \beta V_i + \Pi(k_{-i}) + \beta V_{-i} + 2(S - M - W)]\frac{\partial \theta^{RD,1}}{\partial k_i} = -\frac{\partial(\Pi + \beta V_i^1)}{\partial k_i}\left(\theta^{RD,1} + \Pi_{-i} + \beta V_{-i}^1 - M\right)$$

$$- \beta\frac{\partial V_{-i}^1}{\partial k_i}\left(\theta^{RD,1} + \Pi_i + \beta V_i^1 - M\right) \tag{29}$$

Which yields, differentiating with respect to β,

$$\frac{\partial^2 \theta^{RD,1}}{\partial k_i \partial \beta}[\Pi(k_i) + \beta V_i^1 + \Pi(k_{-i}) + \beta V_{-i}^1 + 2(S - M - W)] + \frac{\partial \theta^{RD,1}}{\partial k_i}\left[V_i^1 + V_{-i}^1 + \beta\frac{\partial V_i^1}{\partial \beta} + \beta\frac{\partial V_{-i}^1}{\partial \beta}\right]$$

$$= -\frac{\partial\left(\Pi_i + \beta V_i^1\right)}{\partial k_i}\left[\frac{\partial \theta^{RD,1}}{\partial \beta} + V_{-i}^1 + \beta\frac{\partial V_{-i}^1}{\partial \beta}\right] - \beta\frac{\partial V_{-i}^1}{\partial k_i}\left[\frac{\partial \theta^{RD,1}}{\partial \beta} + V_i^1 + \beta\frac{\partial V_i^1}{\partial \beta}\right]$$

$$-\left[\beta\frac{\partial^2 V_i^1}{\partial k_i \partial \beta} + \frac{\partial V_i^1}{\partial k_i}\right](\theta^{RD,1} + \Pi(k_{-i}) + \beta V_{-i} - M) - \left[\beta\frac{\partial^2 V_{-i}^1}{\partial k_i \partial \beta} + \frac{\partial V_{-i}^1}{\partial k_i}\right](\theta^{RD,1} + \Pi(k_i) + \beta V_i - M)$$

Since the two ethnic groups are symmetric,

$$V_i^1 = V_{-i}^1 \quad \text{and} \quad \frac{\partial V_i^1}{\partial \beta} = \frac{\partial V_{-i}^1}{\partial \beta}$$

Moreover it is clear from equation (16) that,

$$-\left[\beta\frac{\partial^2 V_i^1}{\partial k_i \partial \beta} + \frac{\partial V_i^1}{\partial k_i}\right](\theta^{RD,1} + \Pi(k_{-i}) + \beta V_{-i} - M) - \left[\beta\frac{\partial^2 V_{-i}^1}{\partial k_i \partial \beta} + \frac{\partial V_{-i}^1}{\partial k_i}\right](\theta^{RD,1} + \Pi(k_i) + \beta V_i - M) < 0$$

Thus, to prove that equation (15) holds, it is sufficient to prove that,

$$-\frac{\partial \theta^{RD,1}}{\partial k_i}\left[2V_i^1 + 2\beta\frac{\partial V_i^1}{\partial \beta}\right] - \left[\frac{\partial \Pi_i + \beta V_i^1}{\partial k_i} + \beta\frac{\partial V_{-i}^1}{\partial \beta}\right]\left[\frac{\partial \theta^{RD,1}}{\partial \beta} + V_i^1 + \beta\frac{\partial V_i^1}{\partial \beta}\right] \leq 0 \tag{30}$$

Since we are in the symmetric case, $\theta^{RD,1} = -(S - W + M + \Pi_i + \beta V_i)/2$. Thus,

$$\frac{\partial \theta^{RD,1}}{\partial \beta} = -\frac{1}{2}[V_i + \beta\frac{\partial V_i}{\partial \beta}]$$

Which yields that condition (30) is equivalent to $-\frac{\partial \theta^{RD,1}}{\partial k_i} - \frac{1}{4}\left[\frac{\partial \Pi_i + \beta V_i^1}{\partial k_i} + \beta\frac{\partial V_{-i}^1}{\partial \beta}\right] \leq 0$. This is indeed the case: combining equation (29) and the expression of $\theta^{RD,1}$, we get that in fact,

$$-\frac{\partial\theta^{RD,1}}{\partial k_i} - \frac{1}{4}\left[\frac{\partial \Pi_i + \beta V_i^1}{\partial k_i} + \beta\frac{\partial V_{-i}^1}{\partial \beta}\right] = 0$$

This concludes the first step of the proof. We now show that when $\theta^{RD,0}$ and $\overrightarrow{V}^0$ satisfy inequalities (15) and (16), then $\overrightarrow{V}^1$ satisfies (16). We have

$$\begin{aligned}\frac{\partial^2 V_i^1}{\partial k_i \partial\beta} &= P(\theta > \theta^{RD,0})\left(\frac{\partial V_i^0}{\partial k_i} + \beta\frac{\partial^2 V_i^0}{\partial k_i\partial\beta}\right) - f_\theta(\theta^{RD,0})\frac{\partial\theta^{RD,0}}{\partial\beta}\beta\frac{\partial V_i^0}{\partial k_i}\\ &- f'_\theta(\theta^{RD,0})\frac{\partial\theta^{RD,0}}{\partial\beta}\frac{\partial\theta^{RD,0}}{\partial k_i}(\Pi_i + \beta V_i + \theta^{RD,0})\\ &- f_\theta(\theta^{RD,0})\frac{\partial^2\theta^{RD,0}}{\partial k_i\partial\beta}(\Pi_i + \beta V_i + \theta^{RD,0})\\ &- f(\theta^{RD,0})\frac{\partial\theta^{RD,0}}{\partial k_i}(\frac{\partial\theta^{RD,0}}{\partial\beta} + V_i + \frac{\partial V_i}{\partial\beta})\end{aligned}$$

Since we have already shown that $\frac{\partial\theta^{RD,0}}{\partial\beta} + V_i + \frac{\partial V_i}{\partial\beta} \geq 0$, simple manipulations of the previous expression show that $\frac{\partial^2 V_i^1}{\partial k_i\partial\beta} > 0$. Similar reasoning shows that $\frac{\partial^2 V_i^1}{\partial k_{-i}\partial\beta} > 0$. This concludes the induction. ■

Proof of Lemma 8. From Lemma 13, without loss of generality, we consider a game for which $W = 0$. As usual, we prove the result by iteratively applying the contraction mapping Φ. In particular we show that if $\overrightarrow{V}^0$ is symmetric and $\theta^{RD,0}$ and $\overrightarrow{V}^0$ satisfy conditions (17) and (18) then, $\theta^{RD,1}$ and $\overrightarrow{V}^1$ also satisfy conditions (17) and (18). Let us show that if $\overrightarrow{V}^1$ satisfies condition (18) then $\theta^{RD,1}$ satisfies inequality (17). Indeed, using the game's symmetry, we have that,

$$2[\Pi(k) + \beta V_i + S - M - W]\frac{\partial\theta^{RD,1}}{\partial k} = -2\frac{\partial\left(\Pi + \beta V_i\right)}{\partial k}\left(\theta^{RD,1} + \Pi_{-i} + \beta V_{-i}^1 - M\right)$$

Which yields, differentiating with respect to k,

$$\frac{\partial^2\theta^{RD,1}}{\partial k^2}[\Pi(k) + \beta V_i^1 + \Pi(k) + S - M - W] + \frac{\partial\theta^{RD,1}}{\partial k}\frac{\partial(\Pi(k) + \beta V_i^1)}{\partial k}$$

$$= -\frac{\partial\left(\Pi(k) + \beta V_i^1\right)}{\partial k}\left[\frac{\partial\theta^{RD,1}}{\partial k} + \frac{\partial\left(\Pi(k) + \beta V_i^1\right)}{\partial k}\right] - \frac{\partial^2\left(\Pi(k) + \beta V_i^1\right)}{\partial k^2}(\theta^{RD,1} + \Pi(k) + \beta V_{-i} - M)$$

Which can be rewritten,

$$\begin{aligned}\frac{\partial^2\theta^{RD,1}}{\partial k^2}[\Pi(k)+\beta V_i^1+\Pi(k)+S-M-W] \quad &= -\beta\tfrac{\partial^2 V_i^1}{\partial k^2}(\theta^{RD,1}+\Pi(k)+\beta V_{-i}-M) \\ &\quad -\left(\tfrac{\partial\left(\Pi(k)+\beta V_i^1\right)}{\partial k}\right)^2-2\tfrac{\partial\theta^{RD,1}}{\partial k}\tfrac{\partial\left(\Pi(k)+\beta V_i^1\right)}{\partial k}\end{aligned} \tag{31}$$

Since we are in the symmetric case, $\theta^{RD,1}=-(S-W+M+\Pi_i+\beta V_i)/2$. Thus,

$$\frac{\partial\theta^{RD,1}}{\partial k}=-\frac{1}{2}\frac{\partial\left(\Pi(k)+\beta V_i\right)}{\partial k}$$

Plugging this into equation (31) we obtain that indeed, $(\partial^2\theta^{RD,1}/\partial k^2)<0$

This concludes the first step of the proof. We now show that when $\theta^{RD,0}$ and $\overrightarrow{V}^0$ satisfy inequalities (15) and (16), then $\overrightarrow{V}^1$ satisfies (16). We have

$$\begin{aligned}\frac{\partial^2 V_i^1}{\partial k^2} &= \beta\frac{\partial^2 V_i^0}{\partial k^2}P(\theta>\theta^{RD,0})-f_\theta(\theta^{RD,0})\frac{\partial\theta^{RD,0}}{\partial k}\frac{\partial(\Pi(k)+\beta V_i^0)}{\partial k} \\ &\quad -f'_\theta(\theta^{RD,0})\left(\frac{\partial\theta^{RD,0}}{\partial k}\right)^2(\Pi(k)+\beta V_i+\theta^{RD,0}) \\ &\quad -f_\theta(\theta^{RD,0})\frac{\partial^2\theta^{RD,0}}{\partial k^2}(\Pi_i+\beta V_i+\theta^{RD,0}) \\ &\quad -f_\theta(\theta^{RD,0})\frac{\partial\theta^{RD,0}}{\partial k}\frac{\partial(\Pi(k)+\beta V_i^0+\theta^{RD,0})}{\partial k}\end{aligned}$$

Simple manipulations of the previous expression show that $(\partial^2 V_i^1/\partial k^2)>0$. This concludes the induction. ■

Proof of Proposition 4. The proof by iteration given in Lemma 2 can be adapted in a straightforward manner. ■

Proof of Lemma 10. The proof of Lemma 6 goes through, replacing ks by zs. ■

Proof of Lemma 9. The proof of Lemma 7 can be adapted in a straightforward manner. ■

Proof of Lemma 11. We have that,

$$V_{i,t}=\mathbf{E}[-W\mathbf{1}_{\theta\le\theta^{RD}}+(\Pi_{i,t}+\beta V_{i,t+1}+\theta)\mathbf{1}_{\theta\ge\theta^{RD}}]$$

Thus,

$$\frac{\partial V_{i,1}}{\partial \Pi_{i,2}} = \beta P(\theta \geq \theta^{RD})\frac{\partial V_{i,2}}{\partial \Pi_{i,2}} - \beta \frac{\partial V_{i,2}}{\partial \Pi_{i,2}}\frac{\partial \theta^{RD}}{\partial \Pi_{i,1}}(\Pi_{i,1} + \beta V_{i,2} + \theta^{RD} + W)f_\theta(\theta^{RD})$$

Noting that $\frac{\partial V_{i,2}}{\partial \Pi_{i,2}} = \frac{\partial V_{i,1}}{\partial \Pi_{i,1}}$, we obtain that,

$$\frac{\frac{\partial V_{i,1}}{\partial k_{i,2}}}{\frac{\partial V_{i,1}}{\partial k_{i,1}}} = \frac{\frac{\partial V_{i,1}}{\partial \Pi_{i,2}}}{\frac{\partial V_{i,1}}{\partial \Pi_{i,1}}} = \beta P(\theta \geq \theta^{RD}) - \beta \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} f_\theta(\theta^{RD})[\Pi_i(k_i) + \beta V_i + \theta^{RD} + W]$$

■

Proof of Lemma 12. From the previous lemma we have that,

$$\rho = \beta P(\theta \geq \theta^{RD}) - \beta \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}} f_\theta(\theta^{RD})[\Pi_i(k_i) + \beta V_i + \theta^{RD} + W]$$

Thus,

$$\begin{aligned}\frac{\partial \rho}{\partial k} = &-\beta f_\theta(\theta^{RD})\frac{\partial \theta^{RD}}{\partial k} - \beta \frac{\partial}{\partial k}\left(\frac{\partial \theta^{RD}}{\partial \Pi_{i,1}}\right) f_\theta(\theta^{RD})[\Pi(k) + \beta V + \theta^{RD} + W] \\ &- \beta f'_\theta(\theta^{RD})\frac{\partial \theta^{RD}}{\partial k}\frac{\partial \theta^{RD}}{\partial \Pi_{i,1}}[\Pi(k) + \beta V + \theta^{RD} + W] - \beta \frac{\partial \theta^{RD}}{\partial \Pi_{i,1}}\left[\frac{\partial \Pi}{\partial k} + \beta \frac{\partial V}{\partial k} + \frac{\partial \theta^{RD}}{\partial k}\right]\end{aligned}$$

Since capital stocks are symmetric, we have that $\theta^{RD} = (M + S - \Pi(k) - \beta V - W)/2$. This implies that

$$\frac{\partial}{\partial k}\left(\frac{\partial \theta^{RD}}{\partial \Pi_{i,1}}\right) = 0$$

and

$$\frac{\partial \Pi}{\partial k} + \beta \frac{\partial V}{\partial k} + \frac{\partial \theta^{RD}}{\partial k} > 0$$

Thus, we have indeed, $\frac{\partial \rho}{\partial k} > 0$. ■