SUPPLEMENT TO "COMPETITIVE CAPTURE OF PUBLIC OPINION"

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1. INTRODUCTION

This is the Online Appendix to "Competitive Capture of Public Opinion." In Section 2 we provide an alternative characterization of communication equilibria in Proposition 1 of the main text, and show that communication equilibria with informed IPs – i.e., such that they observe the underlying state after exerting capture effort but prior to selecting a message – still satisfy the conditions of Proposition 1.

In Section 3 we provide necessary and sufficient conditions for strategic substitutability of capture efforts under Assumption I – i.e., no complementarities in the contest success function $\pi_i(r, l)$. In Section 4 we study several source attributes and clarify when they are vertical or horizontal. We first study audience ideology and show that R (L) wants to fire up its base if its utility is an increasing and convex transformation of the odds of a high (low) state. Therefore, if both IPs have congruent preferences – so that either both want to fire-up-their-base or moderate-the-opposition – then audience ideology is a horizontal attribute: for example, a FOSD increase in citizens priors would increase R's capture incentives but reduce those of L. We then show that the quality of information of honest coverage is generically not a vertical attribute.

Section 5 establishes the existence of pure-strategy capture equilibria with multiple sources. Section 6 describes basic properties of citizen behavior when a fraction of citizens sort according to each source's instrumental value of information. We also show that media markets may become less informative if the demand for information increases. Section 7 provides a complete treatment of capture with naive citizens. Finally, Section 8 studies preference heterogeneity (as opposed to belief heterogeneity) and shows that citizen sorting is robust to this conceptualization of ideology.

2. COMMUNICATION EQUILIBRIA

The following proposition describes the bounds $\overline{\lambda}$ and $\underline{\lambda}$ in Proposition 1 of the main text in terms of bounds on a p-citizen's posterior belief.

PROPOSITION 7: *For every $p-$citizen, the maximum and minimum equilibrium posteriors $\overline{\mu}(p)$ and $\underline{\mu}(p)$ satisfy*

$$\int_{\overline{\mu}(p)}^{1} \overline{F}_H(\mu;p)d\mu = \frac{\pi_R(r,l)}{\pi_H(r,l)}\left(\overline{\mu}(p) - p\right),$$

$$\int_{0}^{\underline{\mu}(p)} F_H(\mu;p)d\mu = \frac{\pi_L(r,l)}{\pi_H(r,l)}\left(p - \underline{\mu}(p)\right).$$

In particular, $\overline{\mu}(p) = \mu_H(\overline{m}^*;p)$ and $\underline{\mu}(p) = \mu_H(\underline{m}^*;p)$ where $\overline{m}^*$ and $\underline{m}^*$ satisfy $\underline{\lambda} = \lambda_H(\underline{m}^*)$ and $\overline{\lambda} = \lambda_H(\overline{m}^*)$ and $\underline{\lambda}$ and $\overline{\lambda}$ are given by (3) and (4).

PROOF: We will express the following equilibrium conditions – see Proposition 1 –

$$\int_{\overline{\lambda}}^{\infty} \left(\lambda - \overline{\lambda}\right) dF_{H,-1}(\lambda) = \frac{\pi_R(r,l)}{\pi_H(r,l)} \left(\overline{\lambda} - 1\right), \tag{1}$$

$$\int_{0}^{\underline{\lambda}} \left(\underline{\lambda} - \lambda\right) dF_{H,-1}(\lambda) = \frac{\pi_L(r,l)}{\pi_H(r,l)} \left(1 - \underline{\lambda}\right), \tag{2}$$

in terms of posterior beliefs $\mu(m;p)$ for $p \in (0,1)$. First, we can write

$$\begin{aligned}\frac{\lambda_H(m) - \lambda_H(\overline{m}^*)}{\lambda_H(\overline{m}^*) - 1} q_{-1}(m) &= \frac{1}{\lambda_H(\overline{m}^*) - 1} q_1(m) - \frac{\lambda_H(\overline{m}^*)}{\lambda_H(\overline{m}^*) - 1} q_{-1}(m) \\ &= \frac{p(1 - \mu_H(\overline{m}^*;p))}{\mu_H(\overline{m}^*;p)) - p} q_1(m) - \frac{\mu_H(\overline{m}^*;p)(1-p)}{\mu_H(\overline{m}^*;p)) - p} q_{-1}(m) \\ &= \left(\frac{\mu_H(m;p)) - \mu_H(\overline{m}^*;p))}{\mu_H(\overline{m}^*;p)) - p}\right) \Omega_H(m;p)\end{aligned}$$

where $\Omega_H(m;p) \equiv q_1(m)p + q_{-1}(m)(1-p)$ is the $p-$citizen's probability density of observing m from honest coverage. Then, (1) can be expressed as

$$\int_{\{m:\mu_H(m;p) \geq \overline{\mu}(p)\}} \left(\mu_H(m;p)) - \overline{\mu}(p)\right) \Omega_H(m;p) dm = \frac{\pi_R(r,l)}{\pi_H(r,l)} \left(\overline{\mu}(p) - 1\right),$$

where $\overline{\mu}(p) \equiv \mu_H(\overline{m}^*;p))$. Integrating by parts,

$$\int_{\overline{\mu}(p)}^{1} \overline{F}_H(\mu;p) d\mu = \frac{\pi_R(r,l)}{\pi_H(r,l)} \left(\overline{\mu}(p) - p\right).$$

where we expressed the result in terms of $\mu = \mu_H(m;p)$. Conversely, from

$$\frac{\lambda_H(\underline{m}^*) - \lambda_H(m)}{1 - \lambda_H(\underline{m}^*)} q_{-1}(m) = \left(\frac{\mu_H(\underline{m}^*;p) - \mu_H(m;p)}{p - \mu_H(\underline{m}^*;p)}\right) \Omega_H(m;p),$$

and letting $\underline{\mu}(p) = \mu_H(\underline{m}^*;p))$, (2) translates, after integrating by parts, to

$$\int_{0}^{\underline{\mu}(p)} F_H(\mu;p) d\mu = \frac{\pi_L(r,l)}{\pi_H(r,l)} \left(p - \underline{\mu}(p)\right).$$

. *Q.E.D.*

In the main text we pointed out that the communication equilibria in Proposition 1 are robust to allowing IPs to condition their message on knowledge of the item's honest coverage. We now formally prove this by considering equilibria in which IPs observe the underlying honest coverage after exerting effort but prior to selecting a message, and show that they still satisfy the conditions of Proposition 1.

PROPOSITION 8: *Fix effort r and l, with $\pi_H(r,l)$>0, and let $(\overline{\lambda}, \underline{\lambda})$ be the unique thresholds derived in Proposition 1. Suppose instead that IPs observe the honest coverage m^j after exerting capture effort but before selecting the coverage. Let $\tau_i^*(\cdot;m^j)$ be i's mixing strategy upon observing realization m^j. Then, in every communication equilibrium, we have*

1. $\cup_{m^j} supp(\tau_R^*(\cdot;m^j)) = \{m : \lambda_H(m) \geq \overline{\lambda}\}$; $\cup_{m^j} supp(\tau_L^*(\cdot;m^j)) = \{m : \lambda_H(m) \leq \underline{\lambda}\}$,
2. *The equilibrium likelihood ratio of message m is given by (2), and the maximum and minimum equilibrium likelihood ratios satisfy* $\max_{m\in\mathcal{M}} \lambda^*(m) = \overline{\lambda}$ *and* $\min_{m\in\mathcal{M}} \lambda^*(m) = \underline{\lambda}$.

PROOF: Suppose that R and L's strategies are $\tau_R(m;m')$ and $\tau_L(m;m')$ so that $\tau_i(m;m')$ is the probability that the i-IP sends m after the source's honest coverage $m^j = m'$. Let $\tilde{\tau}_i(m;m')$ be citizens' assessments of these strategies. Then the perceived likelihood ratio $\lambda(m) \equiv \frac{\Pr[m|\theta=1]}{\Pr[m|\theta=0]}$ is

$$\lambda(m) = \frac{\pi_H(r,l)q_1(m) + \pi_R(r,l)\displaystyle\int \tilde{\tau}_R(m;m')q_1(m')dm' + \pi_L(r,l)\displaystyle\int \tilde{\tau}_L(m;m')q_1(m')dm'}{\pi_H(r,l)q_{-1}(m) + \pi_R(r,l)\displaystyle\int \tilde{\tau}_R(m;m')q_{-1}(m')dm' + \pi_L(r,l)\displaystyle\int \tilde{\tau}_L(m;m')q_{-1}(m')dm'}. \tag{3}$$

The difference between a $p-$citizen's posterior after observing m and m''still satisfies

$$\mu(m;p) - \mu(m'';p) = (\lambda(m) - \lambda(m''))\frac{p(1-p)}{(1-p+p\lambda(m))(1-p+p\lambda(m''))}.$$

Let

$$V_i(m) \equiv \int_0^1 v_i(\mu(m;p))\,dF_p(p) = \int_0^1 v_i\left(\frac{p\lambda(m)}{1-p+p\lambda(m)}\right)dF_p(p). \tag{4}$$

If $\tau_i(m;m')$ are IPs actual strategies, then IPs' optimality requires that if $m_1, m_2 \in supp\,\tau_i(\cdot,m')$ then $V_i(m_1) = V_i(m_2)$, $i \in \{L,R\}$. By the same argument for the case in which IPs do not observe the honest coverage, this requires that $\lambda(m_1) = \lambda(m_2)$.

Let $\lambda^*(m)$ be the equilibrium likelihood ratio of message m with $\widetilde{\lambda} = \max_{m\in\mathcal{M}} \lambda^*(m)$ Note that (i) $V_R(m)$ in (4) is strictly increasing in $\lambda(m)$ while $V_L(m)$ in (4) is strictly decreasing in $\lambda(m)$; and (ii) if $\tau_R(m;m') = \tau_L(m;m') = 0$ for all $m' \in \mathcal{M}$ then (3) implies $\lambda(m) = \lambda_H(m)$. Therefore, from (i) we must have that if $m \in supp(\tau_R^*(\cdot,m'))$ then $\lambda^*(m) = \widetilde{\lambda}$ while (ii) implies that $m \in supp(\tau_R^*(\cdot,m'))$ only if $\lambda_H(m) \geq \widetilde{\lambda}$. Finally, we reach a contradiction if $m \notin \cup_{m^j=m'} supp(\tau_R^*(\cdot;m'))$ and $\lambda_H(m) > \widetilde{\lambda}$ as then we have $\lambda^*(m) = \lambda_H(m) > \widetilde{\lambda} = \max_{m\in\mathcal{M}} \lambda^*(m)$. Therefore, we must have $\cup_{m^j} supp(\tau_R^*(\cdot;m^j)) = \{m : \lambda_H(m) \geq \widetilde{\lambda}\}$ and $m \in supp(\tau_R^*(\cdot,m'))$ iff $\lambda_H(m) \geq \widetilde{\lambda}$. We can apply a similar argument to L by defining $\tilde{\underline{\lambda}} = \min_{m\in\mathcal{M}} \lambda^*(m)$. Then again we reach a contradiction if $m \notin \cup_{m^j=m'} supp(\tau_L^*(\cdot;m'))$ and $\lambda_H(m) < \tilde{\underline{\lambda}}$ as then we must have $\lambda^*(m) = \lambda_H(m) < \tilde{\underline{\lambda}} = \min_{m\in\mathcal{M}} \lambda^*(m)$. Therefore, we must have $\cup_{m^j} supp(\tau_L^*(\cdot;m^j)) = \{m : \lambda_H(m) \leq \tilde{\underline{\lambda}}\}$.

We now show that $\widetilde{\lambda} = \overline{\lambda}$ and $\tilde{\underline{\lambda}} = \underline{\lambda}$. Looking at $\widetilde{\lambda}$, we can rewrite (3) for all m such that $\lambda_H(m) \geq \widetilde{\lambda}$

$$\frac{\pi_R(r,l)}{\pi_H(r,l)}\left(\widetilde{\lambda}\int_{\mathcal{M}} \tau_R(m;m')q_{-1}(m')dm' - \int_{\mathcal{M}} \tau_R(m;m')q_1(m')dm'\right) = \left(\lambda_H(m) - \widetilde{\lambda}\right)q_{-1}(m),$$

and integrating over all $\left\{m : \lambda_H(m) \geq \widetilde{\lambda}\right\}$ and noting that

$$\int_{\left\{m:\lambda_H(m)\geq\widetilde{\lambda}\right\}} \int_{\mathcal{M}} \tau_R(m;m')q_\theta(m')dm'dm = \int_{\mathcal{M}} \left(\int_{\left\{m:\lambda_H(m)\geq\widetilde{\lambda}\right\}} \tau_R(m;m')dm\right) q_\theta(m')dm' =$$
$$= \int_{\mathcal{M}} q_\theta(m')dm' = 1$$

gives

$$\frac{\pi_R(r,l)}{\pi_H(r,l)}\left(\widetilde{\lambda}-1\right) = \int_{\widetilde{\lambda}}^{\infty} \left(\lambda - \widetilde{\lambda}\right) dF_{H,-1}(\lambda),$$

which is the same as (3) which uniquely defines $\overline{\lambda}$. Therefore, $\widetilde{\lambda} = \overline{\lambda}$. A similar argument applied to $\left\{m : \lambda_H(m) \leq \underline{\tilde{\lambda}}\right\}$ shows that $\underline{\tilde{\lambda}} = \underline{\lambda}$. *Q.E.D.*

3. NECESSARY AND SUFFICIENT CONDITIONS FOR STRATEGIC SUBSTITUTABILITY

In Proposition (3) in the main text we showed that capture of a single news source is a game in strategic substitutes as long as there are no interaction effects in $\pi_i(r,l)$ – i.e., if Assumption I holds – and increased capture by one IP does not reduce the odds of capture by the other IP relative to honest coverage – i.e., if Assumption II holds. These assumptions rely solely on properties of the contest success functions as they are independent of the characteristics of the news source and of its audience. Expressing marginal success probabilities as $\frac{\partial \pi_L}{\partial r} = -\alpha \frac{\partial \pi_R}{\partial r}$ and $\frac{\partial \pi_H}{\partial R} = -(1-\alpha)\frac{\partial \pi_R}{\partial r}$, as well as $\frac{\partial \pi_R}{\partial l} = -\beta \frac{\partial \pi_L}{\partial l}$ and $\frac{\partial \pi_H}{\partial l} = -(1-\beta)\frac{\partial \pi_L}{\partial l}$, Assumption II can be equivalently expressed in terms of limits on the crowding-out effect of capture α and β,

$$\frac{\partial}{\partial r}\left(\frac{\pi_L}{\pi_H}\right) \geq 0 \Leftrightarrow \alpha \leq \frac{\pi_L}{1-\pi_R},$$
$$\frac{\partial}{\partial l}\left(\frac{\pi_R}{\pi_H}\right) \geq 0 \Leftrightarrow \beta \leq \frac{\pi_R}{1-\pi_L}.$$

In words, the crowding-out effect of one IP on the other IP's success probability must be sufficiently small, with this upper limit based only on the success probabilities of both IPs. Intuitively, increasing capture by either IP must have a smaller *business-stealing effect* than the effect on the probability of *aggregate capture*. If $\pi_R(r,l) = r$ and $\pi_L(r,l) = l$, then this is always satisfied for all capture levels, as $L/R-$capture only reduces honest reporting, so that $\alpha = \beta = 0$.

We now generalize this insight and show that strategic substitutability holds under more general conditions. We provide necessary and sufficient conditions for capture of a single news item to be a game in strategic substitutes expressed in terms of bounds on the crowding-out effect of capture.

PROPOSITION 9: *Suppose that Assumption I holds. For each* $(r,l;\tilde{r},\tilde{l}) \in (X_R \times X_L)^2$ *with associated sequentially-rational thresholds* $\underline{\lambda}$ *and* $\overline{\lambda}$*, define for* $i \in \{r,l\}$*,*[1]

$$M^i_{\overline{\lambda}} \equiv V_i'(\overline{\lambda})(\overline{\lambda}-1)\frac{\left|\dfrac{\partial \pi_R}{\partial i}+\dfrac{\partial \pi_H}{\partial i}\overline{F}_H(\overline{\lambda};p_i)\right|}{\dfrac{\pi_R}{\pi_H}+\overline{F}_{H,-1}(\overline{\lambda};p_i)}, \tag{5}$$

$$M^i_{\underline{\lambda}} \equiv V_i'(\underline{\lambda})(1-\underline{\lambda})\frac{\left|\dfrac{\partial \pi_L}{\partial i}+\dfrac{\partial \pi_H}{\partial i}F_H(\underline{\lambda};p_i)\right|}{\dfrac{\pi_L}{\pi_H}+F_{H,-1}(\underline{\lambda};p_i)}, \tag{6}$$

and $\kappa_i \equiv \frac{M^i_{\overline{\lambda}}}{M^i_{\underline{\lambda}}}$*. If* $\alpha \equiv -\frac{\partial \pi_L/\partial r}{\partial \pi_R/\partial r}$ *(*$\beta \equiv -\frac{\partial \pi_R/\partial l}{\partial \pi_L/\partial l}$*) is the crowding-out effect of* R *(*L*) on* $L's$ *(*$R's$*) winning probability, then capture of a news item is a game in strategic substitutes if and only if for all* $(r,l;\tilde{r},\tilde{l}) \in (X_R \times X_L)^2$ *we have*

$$\alpha \leq \frac{\pi_L + \dfrac{1}{\kappa_L}(1-\pi_L)}{1-\pi_R+\dfrac{1}{\kappa_L}\pi_R}, \tag{7}$$

$$\beta \leq \frac{\pi_R+\kappa_R(1-\pi_R)}{1-\pi_L+\kappa_R\pi_L}. \tag{8}$$

In particular, if $\kappa_R \geq 1$ *and* $\kappa_L \leq 1$ *for all* $(r,l;\tilde{r},\tilde{l}) \in (X_R \times X_L)^2$ *then capture is a game of strategic substitutes regardless of the size of the crowding out effect* α *and* β*.*

PROOF: Let $W_R(r,l;\tilde{r},\tilde{l}) - C_R(r)$ and $W_L(r,l;\tilde{r},\tilde{l}) - C_L(l)$ be R and L's expected utility when they covertly invest r and l in capturing the item, followed by a sequentially rational reporting strategy where citizens anticipate capture $\tilde{r}$ and $\tilde{l}$ -see (7) in the main text. Then, considering for example the R-IP, we have

$$\frac{\partial W_R(r,l;\tilde{r},\tilde{l})}{\partial r} = \frac{\partial \pi_R(r,l)}{\partial r}V_i(\overline{\lambda}) + \frac{\partial \pi_L(r,l)}{\partial r}V_i(\underline{\lambda}) + \frac{\partial \pi_H(r,l)}{\partial r}\mathbb{E}_H\left[V_i(\lambda);p_R\right].$$

as citizens' interpretation of messages only depends on the expected level of capture $(\tilde{r},\tilde{l})$ rather than the actual level (r,l). Consider the change in $R's$ incentives to increase r when citizens (correctly) anticipate a higher capture level by L

$$\left.\frac{\partial^2 W_R(r,l;\tilde{r},\tilde{l})}{\partial r \partial l}\right|_{l=\tilde{l}} + \left.\frac{\partial^2 W_R(r,l;\tilde{r},\tilde{l})}{\partial r \partial \tilde{l}}\right|_{l=\tilde{l}}$$
$$= \underbrace{\left.\frac{\partial^2 \pi_R(r,l)}{\partial r \partial l}\right|_{l=\tilde{l}}\left[V_R(\overline{\lambda}) - \mathbb{E}_H\left[V_R(\lambda);p_R\right]\right]}_{C_{\overline{\lambda}}}$$

[1]To improve exposition, we omit the dependence of functions on $(r,l;\tilde{r},\tilde{l})$ when this dependence is clear.

$$+ \underbrace{\left.\frac{\partial \pi_L^2(r,l)}{\partial r \partial l}\right|_{l=\tilde{l}} \left[V_R(\underline{\lambda}) - \mathbb{E}_H\left[V_R(\lambda); p_R\right]\right]}_{C_{\underline{\lambda}}}$$

$$+ \underbrace{\left[\left.\frac{\partial \pi_R(r,l)}{\partial r}\right|_{l=\tilde{l}} + \left.\frac{\partial \pi_H(r,l)}{\partial r}\right|_{l=\tilde{l}} \overline{F}_H(\overline{\lambda}; p_R)\right] V_R'(\overline{\lambda}) \frac{\partial \overline{\lambda}}{\partial \tilde{l}}}_{I_{\overline{\lambda}}}$$

$$+ \underbrace{\left[\left.\frac{\partial \pi_L(r,l)}{\partial r}\right|_{l=\tilde{l}} + \left.\frac{\partial \pi_H(r,l)}{\partial r}\right|_{l=\tilde{l}} F_H(\underline{\lambda}; p_R)\right] V_R'(\underline{\lambda}) \frac{\partial \underline{\lambda}}{\partial \tilde{l}}}_{I_{\underline{\lambda}}}$$

The first two terms ($C_{\overline{\lambda}}$ and $C_{\underline{\lambda}}$) capture the complementarities in the contest success function holding constant citizens' beliefs of the levels of capture. Term $C_{\overline{\lambda}}$ represents the second-order marginal effect on $R's$ winning probability weighted by the gain from replacing the honest reporting, while $C_{\underline{\lambda}}$ captures the same effect coming from $L's$ winning probability weighted by the loss to R when L wins and replaces the honest reporting's message. The last two terms ($I_{\overline{\lambda}}$ and $I_{\underline{\lambda}}$) are the informational effects on R's incentives: they represent the change in R's marginal gain that derive solely from the change in citizens' beliefs, and it balances the utility change from inducing the favorable message $\overline{\lambda}$ multiplied by its marginal likelihood (term $I_{\overline{\lambda}}$) with the change when the unfavorable message $\underline{\lambda}$ is induced, multiplied by its marginal likelihood (term $I_{\underline{\lambda}}$).

It is clear that the nature of the contest success function (in particular the sign of $\frac{\partial^2 \pi_i(r,l)}{\partial r \partial l}$) affects the variation in R's incentives with L's anticipated capture. To concentrate on the interactions that are purely informational, we adopt Assumption I so that $\frac{\partial^2 \pi_i(r,l)}{\partial r \partial l} = 0$ for $i \in \{R, L, H\}$ (making $C_{\overline{\lambda}}$ and $C_{\underline{\lambda}}$ identically zero).

For given $(r, l; \tilde{r}, \tilde{l})$, with associated sequentially-rational thresholds $\underline{\lambda}$ and $\overline{\lambda}$,[2] strategic substitutability requires that

$$\left[\left.\frac{\partial \pi_R}{\partial r}\right|_{l=\tilde{l}} + \left.\frac{\partial \pi_H}{\partial r}\right|_{l=\tilde{l}} \overline{F}_H(\overline{\lambda})\right] V_R'(\overline{\lambda}) \frac{\partial \overline{\lambda}}{\partial \tilde{l}} + \left[\left.\frac{\partial \pi_L}{\partial r}\right|_{l=\tilde{l}} + \left.\frac{\partial \pi_H}{\partial r}\right|_{l=\tilde{l}} F_H(\underline{\lambda})\right] V_R'(\underline{\lambda}) \frac{\partial \underline{\lambda}}{\partial \tilde{l}} \leq 0, \tag{9}$$

$$\left[\left.\frac{\partial \pi_R}{\partial l}\right|_{r=\tilde{r}} + \left.\frac{\partial \pi_H}{\partial l}\right|_{r=\tilde{r}} \overline{F}_H(\overline{\lambda})\right] V_L'(\overline{\lambda}) \frac{\partial \overline{\lambda}}{\partial \tilde{r}} + \left[\left.\frac{\partial \pi_L}{\partial l}\right|_{r=\tilde{r}} + \left.\frac{\partial \pi_H}{\partial l}\right|_{r=\tilde{r}} F_H(\underline{\lambda})\right] V_L'(\underline{\lambda}) \frac{\partial \underline{\lambda}}{\partial \tilde{r}} \leq 0, \tag{10}$$

Differentiating $\overline{\lambda}$ and $\underline{\lambda}$ in Proposition 1.3 of the main text we have for $i \in \{r, l\}$,

$$\frac{\partial \overline{\lambda}}{\partial \tilde{i}} = -\frac{\overline{\lambda} - 1}{\frac{\pi_R}{\pi_H} + \overline{F}_{H,-1}(\overline{\lambda})} \left.\frac{\partial}{\partial i}\left(\frac{\pi_R}{\pi_H}\right)\right|_{i=\tilde{i}}$$

$$\frac{\partial \underline{\lambda}}{\partial \tilde{i}} = -\frac{1 - \underline{\lambda}}{\frac{\pi_L}{\pi_H} + \overline{F}_{H,-1}(\underline{\lambda})} \left.\frac{\partial}{\partial i}\left(\frac{\pi_L}{\pi_H}\right)\right|_{i=\tilde{i}}$$

[2] To improve exposition, we omit the arguments of functions when these arguments are clear from the context.

Replacing these expressions in (9) and (10) and using the definition of $M^i_{\overline{\lambda}}$ and $M^i_{\underline{\lambda}}$ in (5) and (6), we obtain that capture of a single news item is a game in strategic substitutes if and only if

$$\frac{-\dfrac{\partial}{\partial l}\left(\dfrac{\pi_R(r,l)}{\pi_H(r,l)}\right)}{\dfrac{\partial}{\partial l}\left(\dfrac{\pi_L(r,l)}{\pi_H(r,l)}\right)} \leq \frac{M^R_{\underline{\lambda}}}{M^R_{\overline{\lambda}}} = \kappa_R, \tag{11}$$

$$\frac{-\dfrac{\partial}{\partial r}\left(\dfrac{\pi_L(r,l)}{\pi_H(r,l)}\right)}{\dfrac{\partial}{\partial r}\left(\dfrac{\pi_R(r,l)}{\pi_H(r,l)}\right)} \leq \frac{M^L_{\overline{\lambda}}}{M^L_{\underline{\lambda}}} = \frac{1}{\kappa_L}. \tag{12}$$

Finally, note that we can express the lhs of (11) and (12) in terms of the crowding-out effect of capture α and β

$$\frac{-\dfrac{\partial}{\partial l}\left(\dfrac{\pi_R(r,l)}{\pi_H(r,l)}\right)}{\dfrac{\partial}{\partial l}\left(\dfrac{\pi_L(r,l)}{\pi_H(r,l)}\right)} = \frac{\beta\pi_H-(1-\beta)\pi_R}{\pi_H+(1-\beta)\pi_L},$$

$$\frac{-\dfrac{\partial}{\partial r}\left(\dfrac{\pi_L(r,l)}{\pi_H(r,l)}\right)}{\dfrac{\partial}{\partial r}\left(\dfrac{\pi_R(r,l)}{\pi_H(r,l)}\right)} = \frac{\alpha\pi_H-(1-\alpha)\pi_R}{\pi_H+(1-\alpha)\pi_R}.$$

Then, (11) and (12) are equivalent to

$$\frac{\beta\pi_H-(1-\beta)\pi_R}{\pi_H+(1-\beta)\pi_L} \leq \kappa_R \Leftrightarrow \beta \leq \frac{\pi_R+\kappa_R(1-\pi_R)}{1-\pi_L+\kappa_R\pi_L},$$

$$\frac{\alpha\pi_H-(1-\alpha)\pi_R}{\pi_H+(1-\alpha)\pi_R} \leq \frac{1}{\kappa_L} \Leftrightarrow \alpha \leq \frac{\pi_L+\dfrac{1}{\kappa_L}(1-\pi_L)}{1-\pi_R+\dfrac{1}{\kappa_L}\pi_R}.$$

Q.E.D.

To explain Proposition 9, consider the capture incentives of, say, R. The crowding-out effect α plays two roles. First, the value of α affects the marginal probability of inducing the favorable interpretation $\overline{\lambda}$ or the unfavorable $\underline{\lambda}$.[3] Second, it dictates how citizens revise their interpretation of messages in light of an increase in $R-$capture. Indeed, while citizens always become more skeptical of high messages, so that $\partial\overline{\lambda}/\partial r \leq 0$, citizens also become skeptical of low messages if α is sufficiently low (in fact, if $\alpha \leq \pi_L/(1-\pi_R)$). Moreover, even if $\alpha > \pi_L/(1-\pi_R)$ we

[3]Indeed, the marginal probability of inducing citizens to interpret the message as $\overline{\lambda}$ or $\underline{\lambda}$ is $\frac{\partial \pi_R}{\partial r} + \frac{\partial \pi_H}{\partial r}\overline{F}_H(\overline{\lambda};p_R) = \frac{\partial \pi_R}{\partial r}\left(1-(1-\alpha)\overline{F}_H(\overline{\lambda};p_R)\right)$ and $\frac{\partial \pi_L}{\partial i} + \frac{\partial \pi_H}{\partial i}F_H(\underline{\lambda};p_R) = -\frac{\partial \pi_R}{\partial r}\left(\alpha+(1-\alpha)F_H(\underline{\lambda};p_R)\right)$.

would have that $R's$ best response is decreasing in L's capture if (7) holds. This conditions ensures, for instance, that the effect of $R-$capture on $R-$lies' is more pronounced than on $L-$lies' – i.e., it ensures that $\partial\overline{\lambda}/\partial r < \partial\underline{\lambda}/\partial r$.

Proposition 9 provides some sufficient conditions for strategic substitutes. First, if $\kappa_R \geq 1$ and $\kappa_L \leq 1$ then the rhs of (7) and (8) are larger than 1. If this holds for all feasible capture levels then capture is a game of strategic substitutes regardless of the size of the crowding out effect α and β. Second, note that $\frac{\pi_L}{1-\pi_R}$ is a lower bound on the rhs of (7) (and $\frac{\pi_R}{1-\pi_L}$ is a lower bound on the rhs of (8)). This confirms that we have a game in strategic substitutes regardless of the properties of the information source and its audience as long as IPs increased capture does not decrease the other IPs success odds relative to honest reporting.

Finally, Proposition 9 also hints to necessary and sufficient conditions for capture to be a game in strategic complements, which would require both inequalities (7) and (8) to be reversed. These conditions are, however, more stringent; for example, these conditions require that $\kappa_R < 1$ and $\kappa_L > 1$ for all feasible capture levels. This is impossible to be satisfied if zero capture is possible for both IPs, as in this case κ_R and κ_L take arbitrarily large and small positive values.

4. SOURCE ATTRIBUTES: AUDIENCE IDEOLOGY AND SOURCE INFORMATIVENESS

In this Section we study conditions under which audience ideology is a horizontal attribute of a source, and show that the quality (informativeness) of honest coverage may not be a vertical attribute.

4.1. *Audience Ideology and IPs incentives: Firing up the Base versus Demobilizing the Opposition*

How an IP's incentives vary with audience priors depends on the priorities of the IP. This is intuitive: an IP which wants to prevent the opposition from coalescing against its preferred policies needs to reach opponents and demobilize them. In contrast, an IP which wants to incite action needs to reach already favorable citizens and further radicalize them. In this section we show that our framework captures this prioritization of audience segments through features of IP's preferences.

To fix language, we say that an IP wants to *fire up the base* if incentives to capture increase when facing a crowd of convinced partisans – i.e., low p for L and high p for R – and an IP wants to *demobilize the opposition* if incentives are stronger with a crowd of opposite partisanship. Formally, R (L) wants to fire up its base if $B_R(r,l;\tilde{r},\tilde{l})(B_L(r,l;\tilde{r},\tilde{l}))$, defined in (9), increases when $F_p(p)$ increases (decreases) in the FOSD sense, with a similar definition for the case in which it wants to demobilize the opposition. Note that, if both IPs have congruent preferences – so that either both want to fire-up-their-base or moderate-the-opposition – then audience ideology is a horizontal attribute: for instance, a FOSD increase in citizens priors would increase R's capture incentives but reduce those of L.

Inspection of (9) shows that audience prior distribution affects capture incentives only through

$$V_i'(\lambda) = \int_0^1 (\partial v_i(\mu(\lambda,p))/\partial\lambda)\, dF_p(p). \tag{13}$$

For $i = R$, $\partial v_R(\mu(\lambda,p))/\partial\lambda$ represents R's marginal payoff from sending a more favorable message to a citizen with prior p and (13) averages this payoff across all citizens. Therefore, R wants to fire up its base if $\partial v_R(\mu(\lambda,p))/\partial\lambda$ increases in p, while it wants to demobilize the opposition if $\partial v_R(\mu(\lambda,p))/\partial\lambda$ decreases in p. Likewise, L wants to fire up its base (demobilize the opposition) if $-\partial v_L(\mu(\lambda,p))/\partial\lambda$ decreases (increases) in p. It follows that in both cases, $i\in\{L,R\}$ wants to fire up its base if and only if $\partial v_i^2(\mu(\lambda,p))/\partial\lambda\partial p\geq 0$. The next proposition links these conditions to the curvature of v_i.

LEMMA 3: *Given a news-source's honest-coverage $F_{H,\theta}$ and its audience's prior distribution F_p, let $[\underline{\mu},\overline{\mu}]$ be the range of posterior beliefs induced if coverage is known to be honest. There are constants $\underline{K}_i$ and $\overline{K}_i$, $i\in\{R,L\}$, with $\underline{K}_R=-\overline{K}_L=K(\overline{\mu})$ and $\overline{K}_R=-\underline{K}_L=K(\underline{\mu})$ where $K(\mu)=\mu/(1-\mu)-(1-\mu)/\mu$, and such that*

I- $i\in\{L,R\}$ wants to fire up its base if $\frac{v_i''(\mu)}{|v_i'(\mu)|}>\underline{K}_i$, $\mu\in[\underline{\mu},\overline{\mu}]$.

II- $i\in\{L,R\}$ wants to demobilize the opposition if $\frac{v_i''(\mu)}{|v_i'(\mu)|}<\overline{K}_i$, $\mu\in[\underline{\mu},\overline{\mu}]$.

PROOF: With $\mu=\mu(\lambda,p)$ to simplify notation, we show that under (I), $\partial^2 v_i(\mu)/\partial\lambda\partial p>0$, while under (II) we have $\partial^2 v_i(\mu)/\partial\lambda\partial p<0$. Differentiating $v_i(\mu)$ twice,

$$\frac{\partial^2 v_i(\mu)}{\partial\lambda\partial p}=v_i''(\mu)\frac{\partial\mu}{\partial\lambda}\frac{\partial\mu}{\partial p}+v_i'(\mu)\frac{\partial^2\mu}{\partial\lambda\partial p}.$$

Using $\frac{\partial\mu}{\partial\lambda}=\frac{p(1-p)}{(\lambda p+1-p)^2}$, $\frac{\partial\mu}{\partial p}=\frac{\lambda}{(\lambda p+1-p)^2}$ and $\frac{\partial^2\mu}{\partial\lambda\partial p}=\frac{1-p-\lambda p}{(\lambda p+1-p)^3}$, we can write

$$\begin{aligned}\frac{\partial^2 v_i(\mu)}{\partial\lambda\partial p}&=v_i''(\mu)\frac{\lambda p(1-p)}{(\lambda p+1-p)^4}+v_i'(\mu)\frac{1-p-\lambda p}{(\lambda p+1-p)^3}\\&=\frac{\lambda p(1-p)}{(\lambda p+1-p)^4}\left(v_i''(\mu)-K(\mu)v_i'(\mu)\right),\end{aligned}$$

with $K(\mu)=\frac{\lambda p}{1-p}-\frac{1-p}{\lambda p}=\frac{\mu}{1-\mu}-\frac{1-\mu}{\mu}$ the difference between the odds of a high state and a low state. As $K(\mu)$ is increasing in μ, we have $K(\mu)\in\left[K(\underline{\mu}),K(\overline{\mu})\right]$ with $[\underline{\mu},\overline{\mu}]$ the range of posteriors of citizens when coverage is known to be honest.

Consider first R. As $v_R'(\mu)>0$, then $\partial^2 v_R(\mu)/\partial\lambda\partial p>0$ if $\min_{\mu\in[\underline{\mu},\overline{\mu}]}\frac{v_R''(\mu)}{v_R'(\mu)}>\max_{\mu\in[\underline{\mu},\overline{\mu}]}K(\mu)=K(\overline{\mu})$ while $\partial^2 v_R(\mu)/\partial\lambda\partial p<0$ if $\max_{\mu\in[\underline{\mu},\overline{\mu}]}\frac{v_R''(\mu)}{v_R'(\mu)}<\min_{\mu\in[\underline{\mu},\overline{\mu}]}K(\mu)=K(\underline{\mu})$. Turning next to L, we have $v_L'(\mu)<0$ so that $\partial^2 v_L(\mu)/\partial\lambda\partial p>0$ if $\min_{\mu\in[\underline{\mu},\overline{\mu}]}\frac{v_L''(\mu)}{|v_L'(\mu)|}>\max_{\mu\in[\underline{\mu},\overline{\mu}]}-K(\mu)=-K(\underline{\mu})$ while $\partial^2 v_L(\mu)/\partial\lambda\partial p<0$ if $\max_{\mu\in[\underline{\mu},\overline{\mu}]}\frac{v_L''(\mu)}{|v_L'(\mu)|}<\min_{\mu\in[\underline{\mu},\overline{\mu}]}-K(\mu)=-K(\overline{\mu})$. *Q.E.D.*

As this lemma shows, if v_i is sufficiently convex, then i is mostly concerned about firing up its base, while if v_i is sufficiently concave, it mostly wants to demobilize the opposition. This is intuitive: for R the gain from raising the beliefs of the public is higher (lower) for those holding very favorable beliefs if v_R is convex (concave). Additional conditions are needed to account for the fact that a higher λ has a smaller (larger) effect on citizens posteriors if citizens hold a higher (lower) prior belief. Notwithstanding, we next show that convexity in the odds of a favorable state are sufficient to guarantee that IPs want to fire up their base.

LEMMA 4: *Suppose that $v_R=g_R(\mu/(1-\mu))$ and $v_L=g_L((1-\mu)/\mu)$, with g_i, $i\in\{L,R\}$, increasing and convex. Then both IPs want to fire up their base.*

PROOF: We can express the odds of the high state as $\mu/(1-\mu) = \lambda p/(1-p)$. Then,

$$\begin{aligned}\frac{\partial^2 v_R(\mu)}{\partial\lambda\partial p} &= \frac{1}{(1-p)^2}\left(g_R''\left(\frac{\lambda p}{1-p}\right)\frac{\lambda p}{1-p} + g_R'\left(\frac{\lambda p}{1-p}\right)\right) \\ &= \frac{1}{(1-p)^2}\left.\frac{d\left(g_R'(x)x\right)}{dx}\right|_{x=\frac{\lambda p}{1-p}}.\end{aligned}$$

If $g_R'(x)x$ is increasing, then R wants to fire up its base, while it wants to demobilize the opposition if $g_R'(x)x$ is decreasing. A sufficient condition for an increasing $g_R'(x)x$ is that g_R is convex. The same analysis applies to L once we observe that

$$\begin{aligned}\frac{\partial^2 v_L(\mu)}{\partial\lambda\partial p} &= \frac{1}{\lambda^2 p^2}\left(g_L''\left(\frac{1-p}{\lambda p}\right)\frac{1-p}{\lambda p} + g_L'\left(\frac{1-p}{\lambda p}\right)\right) \geq 0 \\ &= \frac{1}{\lambda^2 p^2}\left.\frac{d\left(g_L'(x)x\right)}{dx}\right|_{x=\frac{1-p}{\lambda p}}.\end{aligned}$$

Q.E.D.

4.2. *Source Informativeness*

Under what conditions do IPs' capture incentives increase when the source becomes more informative? In other words, when is the quality of honest coverage a vertical attribute of a source? To answer this question, we consider a news source with an audience of fixed size. The direct effect of a more informative source depends on the change in the highest and lowest credible messages, but also on how each IP's payoff depends on the equilibrium informativeness of honest coverage.

To see this, consider the marginal return to R from increasing capture

$$\frac{\partial W_R(r,l;\tilde{r},\tilde{l})}{\partial r} = \frac{\partial \pi_R(r,l)}{\partial r}V_R(\overline{\lambda}) + \frac{\partial \pi_L(r,l)}{\partial r}V_R(\underline{\lambda}) + \frac{\partial \pi_H(r,l)}{\partial r}\mathbb{E}_H\left[V_R(\lambda);p_R\right], \tag{14}$$

when citizens anticipate capture levels $(\tilde{r},\tilde{l})$ – which determine $\overline{\lambda}$, $\underline{\lambda}$ and $\mathbb{E}_H\left[V_R(\lambda);p_R\right]$. If honest coverage becomes more Blackwell-informative, then for the same anticipated capture levels, $\overline{\lambda}$ increases and $\underline{\lambda}$ decreases – see Lemma 1.3.[4] Focusing on the first two terms of (14) and noting that $\frac{\partial \pi_R(r,l)}{\partial r} > 0 > \frac{\partial \pi_L(r,l)}{\partial r}$, we see that $V_R(\overline{\lambda})$ increases and $V_R(\underline{\lambda})$ so that $\frac{\partial W_R(r,l;\tilde{r},\tilde{l})}{\partial r}$ increases.

The difficulty lies in evaluating the change in $\mathbb{E}_H\left[V_R(\lambda);p_R\right]$. There are two main difficulties in signing this change. First, the equilibrium message under honest coverage is not necessarily Blackwell-more informative – see Section 4.2.1 below – making it hard to assess $\mathbb{E}_H\left[V_R(\lambda);p_R\right]$ even if all players (citizens and IPs) share the same prior and $V_R(\lambda)$ is convex/concave. Second, even if the honest message leads in equilibrium to more dispersed (in the monotone convex order) posteriors for a $p-$citizen, the $p-$citizens's posteriors may not be more dispersed if their likelihood is being evaluated by R who has a different prior belief p_R – see, e.g., Alonso and Câmara (2016) for an analysis of how the dispersion of beliefs under one prior can be expressed in terms of a different prior when the information structure is commonly known). We expand on the first difficult in the following Section.

[4]In fact, Lemma 1.3 shows that the equilibrium message is more Blackwell-informative.

4.2.1. *Equilibrium informativeness of honest coverage*

Let M_H^j be the equilibrium citizens' interpretation of messages from the honest coverage of source $j \in \{X, Y\}$ given fixed levels of capture (r, l). We now show by example that it is not true that M_H^Y is Blackwell more informative than M_H^X if Y is Blackwell-more informative than X. To see this, consider the distribution of a $p-$citizen's equilibrium posteriors induced by M_H^j,

$$\tilde{F}_M^j(\mu) = \begin{cases} 0 & \text{if } \mu < \underline{\mu}_j, \\ F_H^j(\mu) & \text{if } \underline{\mu}_j \leq \mu < \overline{\mu}_j, \\ 1 & \text{if } \mu \geq \overline{\mu}_j. \end{cases}$$

Let $\Delta^M(\mu) = \int_0^\mu \tilde{F}_M^Y(s) - \tilde{F}_M^j(s)ds$. We now show that we can have $\Delta^M(\mu) < 0$ for some $\mu \in [0,1]$ even if Y is Blackwell-more informative than X. This implies that posteriors under M_H^Y are not more dispersed (in the monotone convex order) than under M_H^X, so M_H^Y is not Blackwell-more informative thanM_H^X. From (26) and noting that the same capture levels are applied to both sources, we have

$$\int_0^{\underline{\mu}_Y} F_H^Y(s)ds = \int_0^{\underline{\mu}_X} \frac{p - \underline{\mu}_Y}{p - \underline{\mu}_X} F_H^X(s)ds,$$

so that

$$\int_{\underline{\mu}_Y}^{\underline{\mu}_X} F_H^Y(s)ds = \int_0^{\underline{\mu}_X} F_H^Y(s)ds - \int_0^{\underline{\mu}_Y} F_H^Y(s)ds = \int_0^{\underline{\mu}_X} \left(F_H^Y(s) - \frac{p - \underline{\mu}_Y}{p - \underline{\mu}_X} F_H^X(\mu) \right) ds$$

This implies that for $\mu \in [\underline{\mu}_X, \overline{\mu}_X)$, we have

$$\begin{aligned} \Delta^M(\mu) &= \int_{\underline{\mu}_Y}^{\underline{\mu}_X} F_H^Y(s)ds + \int_{\underline{\mu}_X}^{\mu} F_H^Y(s) - F_H^X(s)ds \\ &= \int_0^{\underline{\mu}_X} \left(F_H^Y(s) - \frac{p - \underline{\mu}_Y}{p - \underline{\mu}_X} F_H^X(\mu) \right) ds + \int_{\underline{\mu}_X}^{\mu} F_H^Y(s) - F_H^X(s)ds = \\ &= \frac{\underline{\mu}_Y - \underline{\mu}_X}{p - \underline{\mu}_X} \int_0^{\underline{\mu}_X} F_H^X(\mu)ds + \int_0^{\mu} F_H^Y(s) - F_H^X(s)ds \end{aligned}$$

The first term is negative whenever $\underline{\mu}_Y < \underline{\mu}_X$ while the second term is non-negative if Y is Blackwell-more informative than X. Therefore, any posterior $\mu \in [\underline{\mu}_X, \overline{\mu}_X)$ such that $\int_0^\mu F_H^Y(s) - F_H^X(s)ds = 0$ would have $\Delta^M(\mu) < 0$.

5. COMPETITIVE CAPTURE AND POLARIZATION ACROSS SOURCES

We explore several equilibrium consequences of competitive capture for an exogenous, possibly heterogeneous, audience for each source – thus abstracting from demand-side effects coming from citizens' sorting. First, we show that the existence of a pure-strategy equilibrium in capture efforts for multiple information sources is guaranteed under similar conditions as in Proposition 2 in the main text. We show this result for a general continuous and convex costs of capture $C_R(r)$ and $C_R(l)$.

PROPOSITION 10—Existence of pure-strategy capture equilibria: *Consider a market with n different news sources. IPs $i \in \{R, L\}$ have (i) continuous utilities $v_i(\mu)$; (ii) continuous and convex costs of capture $C_R(r)$ and $C_R(l)$ with $r \in \Pi_{j=1}^n X_R^j$, and $l \in \Pi_{j=1}^n X_L^j$; and (iii) for each source $j \in \{1, ..., n\}$, the probability of state $S^j = i$, $\pi_i^j(r_j, l_j)$, is continuous and concave in r_j and concave in l_j with $\pi_H^j(r_j, l_j) > 0$ for $r_j \in X_R^j$, $l_j \in X_L^j$. Then, there is an equilibrium with pure-strategies capture efforts (r^*, l^*).*

PROOF: Suppose that R selects $r = (r_j)_{j=1}^n$; L selects $l = (l_j)_{j=1}^n$; and citizens have an assessment of IPs' capture strategies $(\tilde{r}, \tilde{l})$ and an assessment of reporting strategies $(\tilde{\tau}_R, \tilde{\tau}_L)$ that is consistent with $(\tilde{r}, \tilde{l})$ – see Proposition 1 in the main text. Then, the payoffs to each IP are $W_R(r, l; \tilde{r}, \tilde{l}) - C_R(r)$ and $W_L(r, l; \tilde{r}, \tilde{l}) - C_L(l)$, where

$$W_R(r, l; \tilde{r}, \tilde{l}) = \sum_{j=1}^{n} \left(\pi_R^j(r_j, l_j) V_R^j(\overline{\lambda}_j) + \pi_L^j(r_j, l_j) V_R^j(\underline{\lambda}_j) + \pi_H^j(r_j, l_j) \mathbb{E}_H^j \left[V_R^j(\lambda); p_R \right] \right),$$

$$W_L(r, l; \tilde{r}, \tilde{l}) = \sum_{j=1}^{n} \left(\pi_R^j(r_j, l_j) V_L^j(\overline{\lambda}_j) + \pi_L^j(r_j, l_j) V_L^j(\underline{\lambda}_j) + \pi_H^j(r_j, l_j) \mathbb{E}_H^j \left[V_L^j(\lambda); p_L \right] \right),$$

with $\overline{\lambda}_j$ and $\underline{\lambda}_j$ satisfying (1) and (2) with $r = \tilde{r}_j$, $l = \tilde{l}_j$, and $V_i^j(\lambda) \equiv \int_0^1 v_i \left(\mu^*(\lambda; p) \right) dF_p^j(p)$. Define i's best-response correspondence given citizens' assessment $(\tilde{r}, \tilde{l})$,

$$\tilde{\Psi}_R(r, l; \tilde{r}, \tilde{l}) \equiv \{ r : W_R(r, l; \tilde{r}, \tilde{l}) - C_R(r) \geq W_R(r', l; \tilde{r}, \tilde{l}) - C_R(r'), r' \in \Pi_{j=1}^n X_R^j \},$$
$$\tilde{\Psi}_L(r, l; \tilde{r}, \tilde{l}) \equiv \{ l : W_L(r, l; \tilde{r}, \tilde{l}) - C_L(l) \geq W_L(r, l'; \tilde{r}, \tilde{l}) - C_L(l'), l' \in \Pi_{j=1}^n X_L^j \},$$

and the belief-consistent best-response correspondence

$$\tilde{\Psi}(r, l) \equiv \{ \tilde{\Psi}_R(r, l; r, l), \tilde{\Psi}_L(r, l; r, l) \}. \tag{15}$$

Note that (r^*, l^*) is a pure-strategy-in-capture-efforts equilibrium if and only if $(r^*, l^*) \in \tilde{\Psi}(r^*, l^*)$. We will apply standard existence results in continuous games with quasiconcave payoffs (see, Debreu (1952), Glicksberg (1952) and Fan (1952)) to show that $\tilde{\Psi}$ has a fixed point.

First, we establish that $W_i(r, l; \tilde{r}, \tilde{l})$ is continuous at each $(r, l; \tilde{r}, \tilde{l})$, and that W_R (W_L) is concave in $r(l)$. For continuity, it suffices to show that $V_i^j(\overline{\lambda}_j)$, $V_i^j(\underline{\lambda}_j)$ and $\mathbb{E}_H^j \left[V_i^j(\lambda); p_i \right]$ are continuous. Define the functions

$$\overline{Q}_j(\lambda) \equiv \frac{\int_\lambda^\infty \overline{F}_{H,-1}^j(\lambda') d\lambda'}{\lambda - 1}; \underline{Q}_j(\lambda) \equiv \frac{\int_0^\lambda F_{H,-1}^j(\lambda') d\lambda'}{1 - \lambda}.$$

Note that $\overline{Q}_j(\lambda) \in \mathbb{R}_{>0}$ is continuous and strictly decreasing for $\lambda > 1$, while $\underline{Q}_j(\lambda) \in \mathbb{R}_{>0}$ is continuous and strictly increasing for $0 \leq \lambda < 1$, thus both possessing a continuous inverse in $\mathbb{R}_{>0}$. The equilibrium thresholds (1-2) imply

$$V_i^j(\overline{\lambda}_j) = V_i^j(\overline{Q}_j^{-1}(\frac{\pi_R^j(r_j, l_j)}{\pi_H^j(r_j, l_j)})),$$

$$V_i^j(\underline{\lambda}_j) = V_i^j(\underline{Q}_j^{-1}(\frac{\pi_L^j(r_j,l_j)}{\pi_H^j(r_j,l_j)})),$$

which are continuous as the composition of continuous functions – as $\pi_H^j(r_j,l_j) > 0$ for $r_j \in X_R^j, l_j \in X_L^j$. Concavity of $W_R(W_L)$ in $r(l)$ follows from concavity of $\pi_i^j(r_j,l_j)$ with respect to $r_j(l_j)$. Therefore, continuity and convexity of $C_R(r)$ and $C_L(l)$ establishes continuity and concavity of $W_R(r,l;\tilde{r},\tilde{l}) - C_R(r)$ and $W_L(r,l;\tilde{r},\tilde{l}) - C_L(l)$.

As X_R^j and X_L^j are compact and convex for each $j = 1,..,n$, continuity of $W_i - C_i$ implies that $\tilde{\Psi}_R(r,l;\tilde{r},\tilde{l})$ and $\tilde{\Psi}_L(r,l;\tilde{r},\tilde{l})$ are upper-hemicontinuous and concavity of $W_R - C_R$ and $W_L - C_L$ imply that they are convex-valued. Upper-hemicontinuity is preserved when restricting attention to the subset $\{(r,l;\tilde{r},\tilde{l}) : l = \tilde{l}\}$ and $\{(r,l;\tilde{r},\tilde{l}) : r = \tilde{r}\}$. Therefore, $\tilde{\Psi}(r,l)$ is non-empty, convex-valued and upper-hemicontinuous and Kakutani's fixed-point theorem guarantees the existence of a fixed point. *Q.E.D.*

5.1. *Source Attributes and Polarization with Interdependent Costs*

We are interested in examining whether competitive capture is conducive to horizontal differentiation. We focus on IP strategies as markers of source polarization. In particular, consider two information sources and let $r = (r_1, r_2)$ and $l = (l_1, l_2)$. Our measure of polarization $\mathcal{P}_I(r,l)$, compares the relative ideological leanings of each source stemming from capture:

$$\mathcal{P}_I(r,l) \equiv \left| \frac{r_1}{l_1} - \frac{r_2}{l_2} \right|.$$

Consider an environment with two information sources and an equilibrium capture (r^*, l^*) with $r_1^*/l_1^* \geq r_2^*/l_2^*$. It is a direct corollary of Proposition 5 that if source 1 experiences a change in an horizontal attribute which favors R, polarization will be higher. It also follows from Proposition 4 that a vertical change can lead as well to an increase in polarization if the indirect effect dominates and therefore one IP does not match the increase in effort by the other.

These results are immediate in the additively separable environment because the effect of changes is circumscribed to source 1 and there is no reason for r_2 or l_2 to change. However, it is also possible to analyze the effect of cost interdependencies that arise naturally as an IP considers deploying limited resources across information sources.

We show below that if costs are interdependent, increases in a horizontal attribute that locally favors the dominant IP spread to produce a more polarized media landscape.

PROPOSITION 11: *Consider the linear-contest model with two information sources and an equilibrium level of capture (r^*, l^*) with $r_1^*/r_2^* > l_1^*/l_2^*$. Suppose that either*

a-both IPs want to fire-up-the-base (demobilize the opposition) and $F_1(p)$ increases (decreases) in the FOSD sense, or

b-$IP-R$'s cost parameters change according to $\tilde{\beta}_1^R$=$\beta_1^R - \delta_1$ and $\tilde{\beta}_2^R = \beta_2^R + \delta_2$, $\delta_1, \delta_2 > 0$, with $\delta_2/\delta_1 = r_1^*/r_2^*$.

Then there is an equilibrium level of capture $(\overline{r}^*, \overline{l}^*)$ such that $\mathcal{P}_I(\overline{r}^*, \overline{l}^*) \geq \mathcal{P}_I(r^*, l^*)$. [5]

Local changes in source characteristics that favor that source's dominant IP spread in equilibrium to widen polarization across sources. To see this, consider case (b) which describes a

[5] Details of the proof are available from the authors.

reduction in the relative capture cost of source 1 by R, keeping invariant the cost of capture under strategy $r^* = (r_1^*, r_2^*)$ to ensure that there are no "wealth" effects.[6] The direct effect of such cost shift leads R to increase capture in source 1 and to decrease it in source 2, holding constant L's strategy. Strategic substitutability implies that the indirect effect generates a reinforcing response: the L decreases capture in source 1 and increases it in source 2. As we had $r_1^*/r_2^* > l_1^*/l_2^*$, both IPs adjust their strategy through a rotation (increasing effort in one source, reducing it in the other) but in opposite directions, increasing both measures of polarization.

Case (a) differs from case (b) as both IPs are directly affected by the change in audience. Consider the case in which both IP want to fire up their base. As the audience of source 1 shifts in favor R, its incentives to capture source 1 increase at the same time that L's weaken. The direct effect of the shift thus leads R to increase capture in source 1, while the L reduces it. The effect on source 2 operates in the opposite direction as both IP equalize expected returns. Strategic substitutability again reinforces both moves as a second order effect. Thus, we have again a rotation in the strategies of IPs that increases media polarization.

Both cases illustrate our main insight in this Section: strategic substitutability is a force towards increased polarization across sources by amplifying local differences in the returns to capture.

6. CITIZENS' SORTING ACROSS INFORMATION SOURCES

In this Section we expand on the analysis in Section 6 of the main text to explore the impact of increasing the fraction of citizens that sort according to instrumental value. In Section 6, we showed that citizens that value information sort across sources (mostly) according to their priors: citizens with extreme priors will prefer the ideologically-aligned source, while if sources share the same informativeness and the likelihood of honest coverage is the same, then all citizens sort monotonically. That is, if some $p-$citizen prefers the left-dominated source, then so do all citizens with $p' \leq p$, while if a $p-$citizen prefers the right-dominated source, then so do all citizens with $p' \geq p$.

This sorting effect is reminiscent of Suen (2004) but we obtain it in a model without filtering in which sources can freely transmit information. In fact, while in Suen (2004) bias is valuable to consumers, in our model the value of information for *all* citizens diminishes with increased capture – see Lemma 2. However, the fact that capture reduces the value of information does not mean that increasing demand for information reduces slant. The following proposition describes a situation in which the opposite is true.

PROPOSITION 12: *Suppose that Assumptions I and II hold;* $v_R = g(\frac{\mu}{1-\mu})$ *and* $v_L = g(\frac{1-\mu}{\mu})$ *with* g *increasing and convex; and there are two symmetric information sources with* $F^1_{H,\theta} = F^2_{H,\theta}\,(= F_{H,\theta})$. *Suppose that for* $\rho \in [0,1)$ *there is an asymmetric equilibrium with* $\bar{\lambda}_1$ $(\underline{\lambda}_2)$ *the highest (lowest) likelihood ratio in media 1 (media 2) which is dominated by* R (L). *Furthermore, there are two equally sized subgroups of citizens* A *and* B*, with priors satisfying*

$$p_k \geq \frac{1}{1+\underline{\epsilon}} > \frac{1}{1+\underline{\lambda}_2} \text{ if } k \in A;\ p_k \leq \frac{1}{1+\bar{\epsilon} < \dfrac{1}{1+\bar{\lambda}_1} \text{ if } k \in B}, \tag{16}$$

[6] More specifically, it rules out the possibility that marginal costs are simultaneously reduced (or increased) for both sources after the change in cost parameters.

and citizens equally likely to consume either source if they do not value information. Then, marginally increasing ρ increases source polarization.[7]

Condition (16) ensures that citizens who value information (a proportion ρ of the population) sort according to group membership – citizens in A patronizing source 1; those in B selecting source 2 – and a marginal increase in ρ will not affect this sorting behavior. The rest of the audience, a fraction $1-\rho$ which do not value information, is spread equally across both sources independent of their prior.

Now consider an increase in ρ. As more citizens now value information, sorting increases: the proportion of citizens in A choosing source 1 and the proportion of citizens in B choosing source 2 both go up. As g is convex enough, Lemma 4 establishes that IPs want to fire up their bases. The sorting described means that R can reach more of its base in source 1 (and less in source 2) and vice versa for L. Both IPs thus rotate their capturing efforts in opposite directions: R increases capture in 1 and reduces it in 2 and L moves in the opposite direction. The fact that capturing efforts are strategic substitutes – guaranteed by Assumptions I and II – further reinforces this dynamic.

As a consequence, as more citizens demand information, the system reacts with more polarization. Slant therefore increases even though the public has higher value for unbiased information. In fact, it is easy to construct examples where citizens are worse off as a result of endogenous sorting if overall capture increases sufficiently. There are limits to this result – for example, we do not consider entry of new information sources as a result of this demand – but it is a cautionary tale on the presumption that slant is driven by lack of interest in knowing the true state of the world.

7. NAIVE CITIZENS

The results we present in the main text rely fundamentally on the rational skepticism of an information source's audience. This begs the question: are these results robust to the presence of unsophisticated citizens? In this section we consider citizens with extreme susceptibility to manipulation. More precisely, we allow for a fraction $1-\gamma<1$ of citizens to be "naive" in that they believe all coverage to be honest. The remainder fraction γ of the audience are fully sophisticated as in previous sections.

Naive and rational citizens interpret the same news λ differently: naive citizens take news at face value and interpret λ literally, while rational citizens are wary of capture and interpret them as $\lambda_\gamma(\lambda)$.[8] The following proposition summarizes the main features of communication equilibria with naive citizens.

PROPOSITION 13: *In the linear-contest model, fix levels of capture r and l, with $r+l<1$, and let $V_i(\lambda) \equiv \int_0^1 v_i\left(\mu^*(\lambda;p)\right) dF_p(p)$ be the expected utility of the $i-$ if citizens interpret the message as λ. There exists a unique equilibrium interpretation of the news by rational citizens $\lambda_\gamma(\lambda)$, with unique $\overline{\lambda}$ and $\underline{\lambda}$, satisfying*

1. *$\lambda_\gamma(\lambda)$ is given by*

$$\lambda_\gamma(\lambda)=\begin{cases} V_L^{-1}\left(V_L(\underline{\lambda})+\frac{1-\gamma}{\gamma}\left(V_L(\underline{\lambda})-V_L(\lambda)\right)\right) & \text{if } \lambda\leq\underline{\lambda}, \\ \lambda & \text{if } \underline{\lambda}<\lambda<\overline{\lambda}, \\ V_R^{-1}\left(V_R(\overline{\lambda})+\frac{1-\gamma}{\gamma}\left(V_R(\overline{\lambda})-V_R(\lambda)\right)\right) & \text{if } \lambda\geq\overline{\lambda}. \end{cases} \tag{17}$$

[7] Details of the proof are available from the authors.

[8] To put it in terms of previous results, Proposition 1 in the main text indicates that when all citizens are rational (i.e., $\gamma=1$), $\lambda_\gamma(\lambda)=\overline{\lambda}$ for $\lambda\geq\overline{\lambda}$ while $\lambda_\gamma(\lambda)=\underline{\lambda}$ for $\lambda\leq\underline{\lambda}$.

2. *The associated* $\overline{\lambda}$ *and* $\underline{\lambda}$ *satisfy*

$$\int_{\overline{\lambda}}^{\infty} \left(\frac{\lambda - \lambda_\gamma(\lambda)}{\lambda_\gamma(\lambda) - 1} \right) dF_{H,-1}(\lambda) = \frac{r}{1-l-r}, \tag{18}$$

$$\int_{0}^{\underline{\lambda}} \left(\frac{\lambda_\gamma(\lambda) - \lambda}{1 - \lambda_\gamma(\lambda)} \right) dF_{H,-1}(\lambda) = \frac{l}{1-l-r} \tag{19}$$

3. $\overline{\lambda}$ *decreases in* l*,* r*, and* γ *while* $\underline{\lambda}$ *is increasing in* l*,* r*, and* γ*. Fixing* $\overline{\lambda}$ *and* $\underline{\lambda}$*, then* $\lambda_\gamma(\lambda)$ *decreases (increases) in* l*,*r*, and* γ *for* $\lambda \geq \overline{\lambda}$ *(*$\lambda \leq \underline{\lambda}$*).*

PROOF: Suppose that the sophisticated citizens' assessments of the reporting strategies of R and L's strategies, expressed in terms of the accepted meaning, are $\tau_R(\lambda)$ and $\tau_L(\lambda)$. Then, the perceived likelihood ratio by sophisticated citizens, $\lambda_\gamma(\lambda) \equiv \frac{\Pr[\lambda|\theta=1]}{\Pr[\lambda|\theta=0]}$, is

$$\lambda_\gamma(\lambda) = \frac{(1-l-r)p_1(\lambda) + r\tau_R(\lambda) + l\tau_L(\lambda)}{(1-l-r)p_{-1}(\lambda) + r\tau_R(\lambda) + l\tau_L(\lambda)}, \tag{20}$$

while i's expected utility from a message that is interpreted as λ is $V_i(\lambda)$. Then, the expected utility of i when sending a message with literal meaning λ is

$$\tilde{V}_i(\lambda) \equiv (1-\gamma)\, V_i(\lambda) + \gamma V_i(\lambda_\gamma(\lambda)).$$

If IPs select $\tau_R(\lambda)$ and $\tau_L(\lambda)$, i's optimality, $i \in \{L, R\}$, requires that if $\lambda, \lambda' \in supp\, \tau_i$, then $\tilde{V}_i(\lambda) = \tilde{V}_i(\lambda')$. We now show that if the distribution $F_H(\lambda)$ is continuous, then (i) $supp\, \tau_i$ is an interval of the form $supp\, \tau_R = [\overline{\lambda}, \lambda_{max}]$ and $supp\, \tau_L = [\lambda_{min}, \underline{\lambda}]$, (ii) $\lambda_\gamma(\overline{\lambda}) = \overline{\lambda}$ and $\lambda_\gamma(\underline{\lambda}) = \underline{\lambda}$, and (iii) λ_γ must satisfy (17) given $\overline{\lambda}$ and $\underline{\lambda}$ for any level of capture.

First, suppose that $F_H(\lambda)$ is a continuous distribution with convex support $supp\, F_H$ and let $\overline{\lambda} \equiv \max\{\lambda : \lambda_\gamma(\lambda) = \lambda, \lambda \in supp\, F_H\}$ be the highest news that sophisticated citizens interpret at face value. Since $\lambda_\gamma(\lambda) \neq \lambda$ implies that $\lambda \in supp\, \tau_R \cup \tau_L$, we must have $\min\{\lambda : \lambda \in supp\, \tau_R\} \leq \overline{\lambda}$. We show that $\min\{\lambda : \lambda \in supp\, \tau_R\} = \overline{\lambda}$. Suppose by contradiction that $\min\{\lambda : \lambda \in supp\, \tau_R\} < \overline{\lambda}$. Then the R obtains utility $\tilde{V}_i(\overline{\lambda}) = V_i(\overline{\lambda})$ from $\overline{\lambda}$, while any $\lambda' \in (\min\{\lambda : \lambda \in supp\, \tau_R\}, \overline{\lambda})$ gives strictly less utility as $\tilde{V}_i(\lambda') \leq V_i(\lambda') < V_i(\overline{\lambda})$. Thus, the R can improve by sending instead $\overline{\lambda}$, thus reaching a contradiction. A similar argument applied to the L implies that $supp\, \tau_L = [\lambda_{min}, \underline{\lambda}]$ and $\lambda_\gamma(\underline{\lambda}) = \underline{\lambda}$. Finally, we obtain (17) by solving for $\lambda_\gamma(\lambda)$ in

$$(1-\gamma)\, V_L(\lambda) + \gamma V_L(\lambda_\gamma(\lambda)) = V_L(\underline{\lambda}) \text{ if } \lambda \leq \underline{\lambda},$$
$$(1-\gamma)\, V_R(\lambda) + \gamma V_R(\lambda_\gamma(\lambda)) = V_R(\overline{\lambda}) \text{ if } \lambda \geq \overline{\lambda}.$$

Note that the equilibrium interpretation (17) depends on $\overline{\lambda}$ and $\underline{\lambda}$. These are pinned down in equilibrium by the condition that each IPs probability of sending each potential lie aggregate to one. Solving for $\tau_R(\lambda)$ and $\tau_L(\lambda)$ in (20)

$$\frac{r}{1-l-r}\tau_R(\lambda) = \frac{\lambda - \lambda_\gamma(\lambda)}{\lambda_\gamma(\lambda) - 1} p_{-1}(\lambda),$$

$$\frac{l}{1-l-r}\tau_L(\lambda) = \frac{\lambda_\gamma(\lambda) - \lambda}{1 - \lambda_\gamma(\lambda)} p_{-1}(\lambda),$$

and integrating these expressions over the respective supports we obtain (18) and (19).

To complete the proof, we write (17) as $\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})$ to make explicit the dependence on $(\overline{\lambda},\underline{\lambda})$ and define

$$\overline{w}(\overline{\lambda}) \equiv \int_{\overline{\lambda}}^{\infty} \frac{\lambda - \lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})}{\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda}) - 1} dF_{H,-1}(\lambda), \tag{21}$$

$$\underline{w}(\underline{\lambda}) \equiv \int_{0}^{\underline{\lambda}} \frac{\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda}) - \lambda}{1 - \lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})} dF_{H,-1}(\lambda). \tag{22}$$

First, we show that $\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})$ is monotonic in $(\overline{\lambda},\underline{\lambda})$. Indeed, as V_R is strictly increasing (and V_L strictly decreasing), then $V_R(\overline{\lambda}) + \frac{1-\gamma}{\gamma}(V_R(\overline{\lambda}) - V_R(\lambda))$ increases in $\overline{\lambda}$ and decreases in γ for any $\lambda > \underline{\lambda}$; similarly, $V_L(\underline{\lambda}) + \frac{1-\gamma}{\gamma}(V_L(\underline{\lambda}) - V_L(\lambda))$ decreases in $\underline{\lambda}$ and increases in γ for any $\lambda < \underline{\lambda}$. Looking at (17) we conclude that, for a fixed value of λ, $\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})$ is non-increasing in $\overline{\lambda}$ and non-decreasing in $\underline{\lambda}$.

Second, we will make use of the fact that $\frac{\lambda-x}{x-1}$ is decreasing in x for $1 < x < \lambda$, while $\frac{x-\lambda}{1-x}$ is decreasing in x for $\lambda < x < 1$. This fact and the monotonicity of $\lambda_\gamma(\lambda;\overline{\lambda},\underline{\lambda})$ in $(\overline{\lambda},\underline{\lambda})$ imply that $\overline{w}(\overline{\lambda})$ in (21) is an strictly decreasing function of $\overline{\lambda}$ with $\overline{w}(\lambda_{max}) = 0$ while $\underline{w}(\underline{\lambda})$ in (22) is an strictly increasing function of $\underline{\lambda}$ with $\underline{w}(\lambda_{min}) = 0$. Furthermore, conditions (18) and (19) translate to $\overline{w}(\overline{\lambda}) = r/(1-r-l)$ and $\underline{w}(\underline{\lambda}) = l/(1-r-l)$. We can then establish uniqueness: As the left hand side of (18) is an strictly decreasing function of $\overline{\lambda}$ and the left hand side of (19) is strictly increasing function of $\underline{\lambda}$, a unique solution to (18-19) is guaranteed for every r and l.

Finally, increasing r or l raises the right hand side of (18) and (19) leading to a lower $\overline{\lambda}$ and higher $\underline{\lambda}$. Likewise, increasing γ lowers both $\overline{w}(\overline{\lambda})$ and $\underline{w}(\underline{\lambda})$, leading to a lower equilibrium $\overline{\lambda}$ and higher $\underline{\lambda}$. *Q.E.D.*

The presence of naive citizens among the public does not qualitatively change our insights regarding message polarization and audience skepticism: the R selects messages with a literal meaning above some $\overline{\lambda}$ while L chooses messages below $\underline{\lambda}$; this results in an increased frequency of extreme messages which, in turn, are not trusted by sophisticated citizens. However, IPs' strategies must now balance the effect of messages on each type of citizen: as naive citizens take messages at face value, selecting messages with more favorable literal meanings must be offset by a less favorable interpretation by sophisticated citizens. This effect is captured in (17) as $\lambda_\gamma(\lambda)$ is decreasing for both $\lambda > \overline{\lambda}$ and for $\lambda < \underline{\lambda}$ – see Figure 1. It follows from (17) that more extreme messages are in this model more heavily discounted by rational citizens and lead to a non-monotonic interpretation: messages whose literal reading would be more favorable are interpreted by sophisticated citizens as having less favorable implications regarding the state of the world.[9]

Another key difference between Proposition 1 and 13 is that, in the presence of naive citizens, communication equilibria can vary with the distribution of priors in the audience. The reason is that each IP's indifference among all potential lies relies on balancing its returns from naive and sophisticated citizens, but an IP's utility from each message interpreted at face value does depend on citizens' priors. This also implies that the highest and lowest trusted news, as given by Part 2 of the Proposition, now vary with the public's distribution of priors.

[9]Chen (2011) provides conditions on the constant bias in the Crawford-Sobel leading example for the existence of communication equilibria in which messages with accepted meaning are interpreted in a non-monotonic way by sophisticated receivers. In our setup, where IPs conflict of interest is extreme, this is a feature of every communication equilibria.

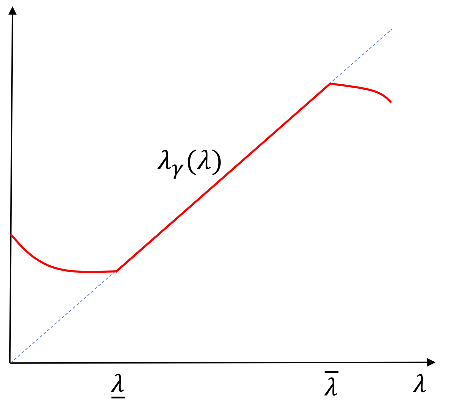

FIGURE 1.—Equilibrium Interpretation by Sophisticated citizens in the presence of Naive citizens.

Finally, increased citizen sophistication (higher γ) makes them trust a smaller set of news – this is in part 3 of Proposition 13. This is intuitive as each IP gains less from pandering to naive citizens. The increased need to convince sophisticated citizens means IPs must reduce the likelihood of sending the most extreme messages and therefore put more weight in more centrist messages.

A key feature of Proposition 13, as shown in part 3, is that increasing the capture level of, say, L, not only reduces $\overline{\lambda}$ and increases $\underline{\lambda}$, but it also affects in a monotonic way the interpretation of the messages by sophisticated citizens: increasing l worsens the interpretation of the messages R sends – by reducing $\lambda_\gamma(\lambda)$ for $\lambda \geq \overline{\lambda}$ – but makes the lies of L more favorable to R – by increasing $\lambda_\gamma(\lambda)$ for $\lambda \leq \underline{\lambda}$. Both effects unambiguously reduce R's marginal gain from capture. Therefore, in this extended model capturing efforts are also strategic substitutes.

PROPOSITION 14: *Suppose that there is a single information source and the probability that R (L) captures the coverage is $r(l)$. Then, for any fraction $\gamma > 0$ of sophisticated citizens, capture efforts are strategic substitutes.*

PROOF: Suppose that citizens anticipate a level of capture $\left(\tilde{r}, \tilde{l}\right)$. R's expected utility when investing r in covertly capturing the source if citizens correctly anticipate R's capture effort is $W_R(r, l; \tilde{r}, \tilde{l})\Big|_{l=\tilde{l}} - C_R(r)$ with

$$W_R(r, l; \tilde{r}, \tilde{l})\Big|_{l=\tilde{l}} = r\tilde{V}_R(\overline{\lambda}) + \tilde{l}\mathbb{E}_{\tau_L}\left[\tilde{V}_R(\lambda); p_R\right] + (1 - r - l)\mathbb{E}_H\left[\tilde{V}_R(\lambda); p_R\right].$$

with

$$\mathbb{E}_H\left[\tilde{V}_R(\lambda); p_R\right] = \overline{F}_H(\overline{\lambda}; p_R)V_R(\overline{\lambda}) + \int_{\lambda_{min}}^{\overline{\lambda}} \left((1-\gamma)V_R(\lambda) + \gamma V_R(\lambda_\gamma(\lambda))\right) dF_H(\lambda; p_R).$$

Therefore, R 's marginal gain from covertly increasing media capture is $B_R(\tilde{r}, \tilde{l}) - C'_R(r) \equiv \frac{\partial W_R(r,l;\tilde{r},\tilde{l})}{\partial r}\Big|_{l=\tilde{l}} - C'_R(r)$ where

$$B_R(\tilde{r}, \tilde{l}) = V_R(\overline{\lambda}) - \mathbb{E}_H\left[V_R(\lambda); p_R\right]$$

$$= \int_{\lambda_{min}}^{\overline{\lambda}} \left(V_R(\overline{\lambda}) - V_R(\lambda)\right) dF_H(\lambda; p_R) \tag{23}$$

$$- (1-\gamma) \int_{\lambda_{min}}^{\underline{\lambda}} \left(V_R(\lambda_\gamma(\lambda)) - V_R(\lambda)\right) dF_H(\lambda; p_R). \tag{24}$$

By increasing capture efforts, R obtains $V_R(\overline{\lambda})$ instead of the utility derived from an honest coverage $\mathbb{E}_H\left[V_R(\lambda); p_R\right]$. Thus, the R gains $V_R(\overline{\lambda}) - V_R(\lambda)$ whenever $\lambda \leq \overline{\lambda}$ and all citizens (including sophisticated ones) interpret the message at face value – this is (23) – except when $\lambda \leq \underline{\lambda}$ and sophisticated citizens discount the news – this is (24).

We now show that $\partial B_R(\tilde{r}, \tilde{l})/\partial \tilde{l} \leq 0$ so the R's incentives to capture decrease with the anticipated level of capture of L. First, part 3 of Proposition 13 shows that $\overline{\lambda}$ decreases with l, so (23) decreases with $\tilde{l}$. Moreover, part 3 of Proposition 13 also shows that increasing l, (a) increases $\lambda_\gamma(\lambda)$ for $\lambda \leq \underline{\lambda}$, and (b) increases $\underline{\lambda}$. Both effects raise the value of the integral in (24), thus decreasing (24). Therefore, increasing $\tilde{l}$ lowers $B_R(\tilde{r}, \tilde{l})$. A similar analysis applied to capture by L shows that $\partial B_L(\tilde{r}, \tilde{l})/\partial \tilde{r} \leq 0$. *Q.E.D.*

This section therefore establishes that our main results, while driven by rational skepticism, are not knife-edge. Even in the presence of a large share of citizens who believe the lies they are fed, strategic and competitive IPs must still consider how sophisticated citizens update, which leads to their efforts being strategic substitutes.

8. SORTING WITH A COMMON PRIOR AND HETEROGENEOUS PREFERENCES

Suppose that all citizens share a common prior p but differ in their payoffs from acting/not-acting: an $\alpha-$citizen obtains $1-\alpha$ if $a = 1$ and $\theta = 1$; α if $a = -1$ and $\theta = -1$; and 0 otherwise. We let $F_\alpha(\alpha)$ be the distribution of α in the audience of the source.

Note that an $\alpha-$citizen will select $a = 1$ whenever her posterior $\mu \geq \alpha$. This implies that if $p < \alpha$, then this citizen selects $a = -1$ in the absence of information and will select $a = 1$ when the equilibrium informational content of the message $\lambda^*(m) \geq \lambda_{crit}(\alpha)$, where

$$\frac{\alpha}{1-\alpha} = \lambda_{crit}(\alpha)\frac{p}{1-p}.$$

So, similar to the case of heterogeneous priors, λ_{crit} is the minimum informational content of a message that will lead a citizen with threshold α to act. If $p > \alpha$ then this citizen selects $a = 1$ in the absence of additional information and will change her decision to $a = -1$ only if $\lambda^*(m) \leq \lambda_{crit}(\alpha)$.

Recall that $F^j_\mu(\mu, p)$ is the distribution over posterior beliefs of a citizen consuming source j, where p is now citizens' common prior. We can derive the value of information for an α-citizen when consuming source j. First, if $p > \alpha$ then

$$I^j(\alpha) \equiv \int_0^\alpha [\alpha(1-\mu) - (1-\alpha)\mu] dF^j_\mu(\mu, p) = \int_0^\alpha (\alpha - \mu) dF^j_\mu(\mu, p) = \int_0^\alpha F^j_\mu(\mu, p) d\mu$$
$$= \int_0^{\lambda_{crit}(\alpha)} F^j(\lambda, p)\frac{p(1-p)}{(1-p+\lambda p)^2} d\lambda,$$

where we made the change of variables $\lambda = \frac{\mu}{1-\mu}\frac{1-p}{p}$ to obtain the last term, and we used $\lambda_{crit}(\alpha) = \frac{\alpha}{1-\alpha}\frac{1-p}{p}$. This follows as the citizen will change her decision from $a = 1$ to $a = -1$

only after observing a message that leads her to a posterior belief $\mu \leq \alpha$ – i.e., a message with $\lambda \leq \lambda_{crit}(\alpha)$. Equivalently, if $p < \alpha$

$$I^j(\alpha) \equiv \int_{\alpha}^{1} [(1-\alpha)\mu - \alpha(1-\mu)] dF^j_{\mu}(\mu, p) = \int_{\alpha}^{1} \overline{F}^j_{\mu}(\mu, p) dp = \int_{\lambda_{crit}(\alpha)}^{\infty} \overline{F}^j(\lambda, p) \frac{p(1-p)}{(1-p+\lambda p)^2} d\lambda.$$

Note that these expressions are identical to the case of common preferences and heterogenous priors if we replace $\lambda_{crit}(p)$ in (18) with $\lambda_{crit}(\alpha)$. In other words, the sorting behavior of a $p'-$citizen in our original model – that approves whenever her posterior exceeds $1/2$ – is the same as an $\alpha-$citizen when all citizens share the same common prior p if

$$\lambda_{crit}(p') = \lambda_{crit}(\alpha) \Rightarrow \alpha = \frac{1}{1 + \frac{(1-p)p'}{(1-p')p}}.$$

Therefore, all our insights on citizens with heterogeneous priors sorting across sources carry over, mutantis mutandi, if we instead assume that they share a common prior but have heterogenous preferences.

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